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Fast phase retrieval with four-quadrant analysis in phase-shifting interferometry with blind phase shifts

Yuanyuan Xu [a], Yawei Wang [a,*], Hao Han [a], Jingye Liu [a], Ying Ji [a], Weifeng Jin [b], Xiaoqing Xu [b]

[a] *Faculty of Science, Jiangsu University, Zhenjiang 212013, China*
[b] *School of Mechanical Engineering, Jiangsu University, Zhenjiang 212013, China*

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ABSTRACT

Phase-shifting interferometry (PSI) is one of the most effective techniques in optical measurement, in which phase retrieval with high efficiency is an important procedure. In this paper, a simple non-iterative method is proposed to extract the generalized phase shift with the four-quadrant analysis in three-frame PSI. In this method, the possible value of the phase shift is firstly worked out with the inner product algorithm, and then a criterion is put forward to accurately determine its principal value within the range $[0, 2\pi]$, based on the change relationship of the interference wave vector in four quadrants. Thus, this method provides a possible method to solve the uncertainty of phase shift existing in some common algorithms. Subsequently, the phase can be retrieved easily without any other measurements. Both simulation and experimental results have fully proved the feasibility and high accuracy of the method. Moreover, it works well on open- and closed-fringed patterns.

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1. Introduction

Phase-shifting interferometry (PSI) is an important technique widely used in optical measurement and microscopy [1,2]. In PSI techniques, phase shift deviation is one of the error sources, causing accuracy reduction of phase retrieval. Therefore, improvement in accuracy of phase shift extraction is a key issue. To address it, many generalized methods [3–17] have been proposed to extract phase shifts from holograms directly.

Phase shift extraction methods can be divided into two broad categories: iterative and non-iterative. The iterative methods [3,4], such as the least squares method [3], can determine the phase shift with high accuracy. But they are very time-consuming since the procedures are repeated many times. For this reason, non-iterative methods [5–17] have been favored for their higher calculating speed and less computational loads. For instance, in Refs. [5,6], a kind of spatial statistical algorithms are proposed to calculate the unknown phase shift, which are based on the phase random condition [5]. In Ref. [7], with the maximum and minimum values of the interference term, an accurate algorithm for phase shift extraction in two-step PSI is proposed. Of course, some other simple and efficient methods, such as the zero difference algorithm [8] and the Euclidean matrix norm algorithm [9], are also introduced to perform the phase shift extraction. Although these non-iterative methods possess their own merits, most of them suffer from the sign ambiguity of the phase shift. Consequently, the phase shift must be restricted to the range of $[0, \pi]$ in advance.

In order to determine the generalized phase shift, its range is required to be extended to the principle range $(0, 2\pi)$. In recent years, some non-iterative methods [10–17] have been reported to achieve it. In Refs. [10–12], a kind of advanced statistical method is proposed to estimate the phase shift. In Refs. [10,11], the sign of the relative phase shift is determined from eight cyclic phase constraint conditions, which are consisted of three holograms. In Ref. [12], based on the spatial frequency relationship between two interference waves, the quadrant sign of phase shift is determined. In these methods, the intensities of the interference waves are required to be measured separately. As a result, the whole processing time is increased and the phase shift exaction appears complicated. Methods reported in Refs. [13–17] are based on other different algorithms to directly retrieve the phase. A significant advantage of these methods is that the phase shift does not need to be known and can take any value inside the range $(0, 2\pi)$. For example, in Ref. [13], a two-step demodulation algorithm based on the self-tuning algorithm is introduced. However, the accuracy of phase shift decreases when the phase shift is far from $\pi/2$. In Ref. [14], the Gram–Schmidt (GS) orthonormalization algorithm is employed to directly measure the phase from two phase-shifted interferograms. This

* Corresponding author.
E-mail address: jszjwyw@sina.cn (Y. Wang).

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algorithm is fast and accurate. In Ref. [15], a demodulating two-step method based on a regularized optical flow method is presented. This method can obtain the phase in a robust and fast way. However, it requires the loaded computations to perform the optical flow analysis. In addition, like the GS algorithm, the phase shift cannot take the singular value of π. In Refs. [16,17], an asynchronous phase-shifting method based on principal component analysis (PCA) method is proposed. This method has some advantages. However, it requires a large number of interferograms to obtain a reliable result.

In this letter, a simple phase shift extraction method for three-frame PSI is proposed. It is based on both the inner product characterization of the intensity difference and the relationship of interference wave vector with the phase shift. In this method, only three phase-shifted interferograms are required, and the phase shift can take any value within the range $[0, 2\pi]$ including the singular value of π. Thus, the sign ambiguity of the phase shift is avoided. After determining the phase shifts, the phase can be retrieved easily with the phase-shifting formula.

2. Method

In order to achieve quantitative phase imaging, the three- or more-frame PSI is usually used by virtue of its high precision and simple operation. For a three-frame generalized PSI, the intensity distribution of each interferogram can be given in the following form:

$$\begin{aligned} I_{kmn} &= \left|O_{mn} + R_{kmn}\right|^2 = \left|O_{mn}\right|^2 + \left|R_{kmn}\right|^2 \\ &\quad + 2\left|O_{mn}\right| \cdot \left|R_{kmn}\right| \cos\left[\phi_{mn} - \delta_k\right] \\ &= a_{mn} + b_{mn}\cos\left[\phi_{mn} - \delta_k\right] \ (\mathrm{k} = 1, 2, 3). \end{aligned} \tag{1}$$

Here, m and n denote the pixel coordinates of rows and columns of an interferogram, respectively. $O_{mn} = \left|O_{mn}\right| \exp\left[j\phi_{omn}\right], \left|O_{mn}\right|$ and ϕ_{omn} are the complex amplitude, amplitude and phase of the object wave, respectively. $R_{kmn} = \left|R_{mn}\right| \exp\left[j\left(\phi_{rmn} + \delta_k\right)\right], \left|R_{mn}\right|$ and ϕ_{rmn} are the corresponding results of the k th frame reference wave, in which the phase shift related to the k th frame, δ_k, is usually assumed to be zero when $k = 1$. $\phi_{mn} = \phi_{omn} - \phi_{rmn}$ is the relative phase difference between the object and reference waves and is required to be measured. The difference between the p th and q th interferograms can be expressed as

$$\begin{aligned} \Delta I_{pq} &= I_{pmn} - I_{qmn} \\ &= 2b_{mn}\sin\left[\phi_{mn} - \frac{\delta_p + \delta_q}{2}\right]\sin\left[\frac{\delta_p - \delta_q}{2}\right] \quad (p, q = 1, 2, 3). \end{aligned} \tag{2}$$

Then, the inner product operator is applied to process ΔI_{pq}. We have

$$\begin{aligned} S_{pq} &= \left\langle \Delta I_{pq} \cdot \Delta I_{pq} \right\rangle = \sum_{m=1}^{M}\sum_{n=1}^{N} 4b_{mn}^2 \sin^2\left[\phi_{mn} - \frac{\delta_p + \delta_q}{2}\right]\sin^2\left[\frac{\delta_p - \delta_q}{2}\right] \\ &= 4sin^2\left[\frac{\delta_p - \delta_q}{2}\right]\sum_{m=1}^{M}\sum_{n=1}^{N} b_{mn}^2 \sin^2\left[\phi_{mn} - \frac{\delta_p + \delta_q}{2}\right]. \end{aligned} \tag{3}$$

If the fringe number in each interferogram is more than one, then the measured phase varies more than 2π (rad) in the observed area. As a result, the following approximation can be applied,

$$\sum_{m=1}^{M}\sum_{n=1}^{N} b_{mn}^2 \sin^2\left[\phi_{mn} - \frac{\delta_p + \delta_q}{2}\right] \approx \sum_{m=1}^{M}\sum_{n=1}^{N} b_{mn}^2 \sin^2\left[\phi_{mn}\right]. \tag{4}$$

Thus, Eq. (3) can be simplified as

$$\begin{aligned} S_{pq} &= 4sin^2\left[\frac{\delta_p - \delta_q}{2}\right]\sum_{m=1}^{M}\sum_{n=1}^{N} b_{mn}^2 \sin^2\left[\phi_{mn}\right] \\ &= C \cdot \left[1 - \cos\left(\delta_p - \delta_q\right)\right] \end{aligned} \tag{5}$$

with $C = 2 \cdot \sum_{m=1}^{M}\sum_{n=1}^{N} b_{mn}^2 \sin^2\left(\phi_{mn}\right)$. From Eq. (5), it is clear shown that S_{pq} changes in cosine form with the relative phase shift $\Delta\delta_{pq} = \delta_p - \delta_q$, and there are only three unknown quantities, namely δ_2, δ_3 and C, in three-frame PSI. According to Eq. (5), there are three quantities as follows:

$$\begin{aligned} &S_{12} = C.\left[1 - \cos\left(\delta_2\right)\right], S_{13} = C.\left[1 - \cos\left(\delta_3\right)\right], \\ &S_{23} = C.\left[1 - \cos\left(\delta_2 - \delta_3\right)\right]. \end{aligned} \tag{6}$$

Since S_{12}, S_{13} and S_{23} can be determined, the values of δ_2 and δ_3 can be calculated. However, there are several solutions corresponding to them according to the property of cosine function. As a result, the sign uncertainty of the phase shift appears. Of course, it also exists in some common algorithms [7–9]. In general, to settle this problem, the phase shifts are restricted to the range of $(0, \pi)$ in advance. Because the cosine form of the phase shift is a monotonic function in this range.

In fact, as long as we can judge whether the value of the phase shift is in the range of $(0, \pi)$ or is in the range of $(\pi, 2\pi)$, we can determine its correct value within the principle range of $(0, 2\pi)$ on the basis of Eq. (6). In this work, based on the analysis of the interference wave vector, we propose a simple method to solve the above problem. The detailed deductions are as follows.

If O, R_1 and A_1 are the object, reference and interference total waves, the vector relationship $A_1 = O + R_1$ is satisfied according to the interference principle. Fig. 1(a) shows the relationship between these optical waves at a special position clearly, where the phase difference between the object and reference waves, ϕ_{or} is equal to $\pi/2$. In addition, the length of each wave represents its amplitude. If a phase shift of δ is introduced to delay the phase of the reference wave, the interference total wave changes and the phase difference is changed to be $\phi_{or} - \delta$. Figs. 1(b) and 1(c) show that R_1 is changed to be R_2 by the aid of the phase shift δ within the ranges of $(0, \pi)$ and $(\pi, 2\pi)$, respectively. As a result, the total wave A_1 is changed to be $A_2 = O + R_2$. From Fig. 1(b), we can find that the amplitude of A_2(or the intensity $I_2 = \left|A_2\right|^2$) is larger than that of A_1(or the intensity $I_1 = \left|A_1\right|^2$) when $0 < \delta < \pi$. While from Fig. 1(c), we found that the amplitude of A_2(or the intensity $I_2 = \left|A_2\right|^2$) is less than that of A_1(or the intensity $I_1 = \left|A_1\right|^2$) when $\pi < \delta < 2\pi$. In fact, we also can compare the phase-shifted interference intensities at another special position, where the phase difference ϕ_{or} is equal to $3\pi/2$. It is not difficult to find that the conclusion obtained in this case is contrary to the above conclusion.

From the above analysis, as long as we can find the special point and then compare the intensities of phase-shifted interferograms at this point, the quadrant range of the phase shift can be determined. Then, with Eq. (6), the unique value of each phase shift can be correctly calculated. In following, we take into consideration the case of $\phi_{or} = \pi/2$.

Here, in order to determine the position of the special point, where the phase difference is equal to $\pi/2$, the interference term of the 1st interferogram is required to be obtained by the filter procedure at first, namely $\widetilde{I}_{1mn} = b_{mn}\cos\phi_{mn}$. Then the Hilbert transform is applied to process $\widetilde{I}_{1mn}$. We have

$$HT(\widetilde{I}_{1mn}) = b_{mn}\sin\phi_{mn}. \tag{7}$$

Obviously, $HT\left(\widetilde{I}_{1mn}\right)$ changes in sine form with the phase. Thus, we can easily find the special point by searching the maximum value of $HT\left(\widetilde{I}_{1mn}\right)$.

It is worth noting that the wave vector analysis above cannot work on the singular case of $\delta = \pi$, since $I_1 = a_{mn} + b_{mn}\cos(\pi/2 - 0)$ and $I_2 = a_{mn} + b_{mn}\cos(\pi/2 - \pi)$ are the same in this case. To settle this problem, we need to further compare the sizes of I_1 and I_2 at other arbitrary point. If the relationship $I_1 = I_2$ is still satisfied, the phase shift δ is equal to 2π, otherwise $\delta = \pi$. Thus, the phase shift can be any value within the range $[0, 2\pi]$.

Once the phase shift of each frame is determined, the phase can be retrieved with the following expression

$$\phi = \arctan\frac{I_3 - I_2 + \left(I_1 - I_3\right)\cos\delta_2 + \left(I_2 - I_1\right)\cos\delta_3}{\left(I_3 - I_1\right)\sin\delta_2 + \left(I_1 - I_2\right)\sin\delta_3}. \tag{8}$$

3. Simulation and discussion

In order to verify the effectiveness of the method proposed above, a series of numerical simulations associated with a spherical wavefront have been carried out. The background intensity and modulation

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