Optics Communications 407 (2018) 193–198

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Scattering cross section in a cylindrical anisotropic layered metamaterial

M.R. Forouzeshfard *, Masoud Mohebbi, Aliyeh Mollaei

Department of Physics, Faculty of Science, Vali-e-Asr University, P.O. Box No: 77139-36417, Rafsanjan, Iran

ARTICLE INFO

Keywords:
Scattering efficiency
ENZ metamaterials
Plasmonics
Hyperbolic media
Invisibility

ABSTRACT

To design a uniaxial anisotropic metamaterial, a layered cylindrical metamaterial is introduced for TE polarization. Unlike to the previous work, which the layers were in radial direction, here the layers are in azimuthal direction. Scattering efficiency for this metamaterial in different frequency is analyzed with solving Maxwell's wave equation. It is observed that in some frequencies when the effective permittivity of the structure goes to zero the scattering efficiency would be negligible. This result approves the previous predictions. It is also found out that the scattering cancellation depends on the relative permittivity of the environmental medium for the cylinder. The finite element simulations are also confirmed the results.

1. Introduction

Cloaking [1–3] and transparency [4–9] are the subjects which attract many interests in all the world during the last decade. One of the approach in cloaking is based on scattering suppression. Epsilon-Near-Zero (ENZ) material are very good candidate for this purpose [9–12].

Introduction of hyperbolic layered metamaterials was a very significant progress to realization of uniaxial ENZ material [13,14]. It was proved that the scattering cross section of a small arbitrary object at the center of a planar layered metamaterial with effective ENZ permittivity can be suppressed provided that dipole approximation be valid [15].

Kim et al.'s are also introduced a cylindrical layered metamaterial for both TE and TM polarization so that for a cylinder cavity made of alternating metal–dielectric structure in radial direction the invisibility condition in a specified frequency is happened when the effective permittivity of the structure goes to zero [16]. Another research also approved the result extracted by Kim et al.'s, however it shows that the frequency of invisibility is also depend on the permittivity of the environment medium [17]. Using graphene-coated nanowire in order to suppress the scattering cross section of a cylindrical cavity is also studied [18].

In this paper we also consider a cylindrical hyperbolic metamaterial, but with layered structure in azimuthal direction for TE polarization. The scattering efficiency of this structure is studied with solving Maxwell's wave equation analytically and the condition for minimizing the scattering cross section is found.

Some finite element simulations are also brought to approve the validation of the analytical solution.

This paper is organized as follows. In Section 2 we solve Maxwell's wave equation for a uniaxial anisotropic cylinder and the scattering cross section for this cylindrical scatterer is found analytically. In Section 3 we proposed a hyperbolic layered metamaterial composed of metal–dielectric layers and its effective permittivities is calculated using effective medium theory. Analyzing the scattering cross section of this metamaterial structure and discussion on the condition of the invisibility is expressed in Section 4. We conclude the result in Conclusion section.

2. Analytical discussion

Consider a long coated cylinder with inner (outer) radius $R_1(R)$ so that the thickness of the shell is $T = R - R_1$. The electric permittivity and permeability for the core is ε_c and μ_c, respectively and the shell is an anisotropic material with permittivity and permeability tensor expressed by

$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \qquad \vec{\mu} = \begin{pmatrix} \mu_r & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_z \end{pmatrix}. \tag{1}$$

The permittivity and permeability of the environment medium is denoted by ε_0, μ_0 as shown in Fig. 1.

An incident plane wave with TE polarization (electric field along z direction) is illuminated to the cylinder normally so that the wave vector is in the x-direction. We are going to find the constraint in which the total scattering cross section of this structure goes to zero. Lorentz–Mie scattering theory is used in order to find the solution of Maxwell's wave

* Corresponding author.
E-mail address: m.forouzeshfard@vru.ac.ir (M.R. Forouzeshfard).

http://dx.doi.org/10.1016/j.optcom.2017.09.009
Received 3 August 2017; Received in revised form 30 August 2017; Accepted 4 September 2017
Available online 22 September 2017
0030-4018/

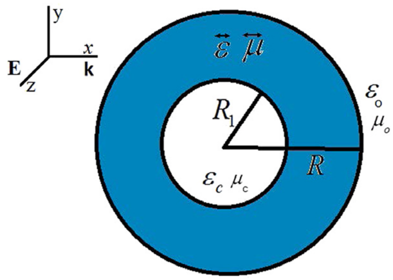

Fig. 1. A cylinder with anisotropic coated shell.

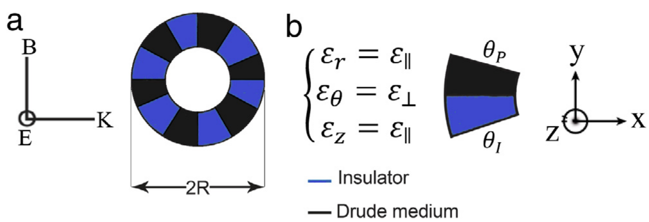

Fig. 2. (a) Structure of a proposed layered metamaterial for TE illumination. (b) The unit cell of the structure in part (a) composed of a layer of Drude medium and an insulator layer.

equation in all three regions. The approach which we apply to find the scattering efficiency is similar to the method in Ref. [19] and [20] but for a TE plane wave illumination. Since for the outside region $r > R$, we have an isotropic medium, so, for the electric field at the outside of the coated cylinder we have [21]

$$E_z{}^{tot} = E_z{}^{i} + E_z{}^{sc}, \tag{2}$$

where in

$$E_z{}^{i} = E_0 e^{ikx} = E_0 \sum_{n=-\infty}^{+\infty} i^n J_n\left(k_0 r\right) e^{in\phi}, \tag{3}$$

is the electric field for the incident wave and E_z^{sc} is the scattered electric field from our structure and can be expanded in cylindrical coordinate as follows [21]

$$E_z^{sc} = E_0 \sum_{n=-\infty}^{+\infty} i^n a_n H_n^{(1)}(kr)\, e^{in\phi}, \tag{4}$$

where $H_n^{(1)}$ is the Hankel function of the first kind of order n; $k = k_0\sqrt{\varepsilon_0\mu_0}$ and $k_0 = \omega/c$.

Electric field in a core layer is as follows [21]

$$E_z{}^{(1)} = E_0 \sum_{n=-\infty}^{+\infty} i^n d_n J_n\left(k_1 r\right) e^{in\phi}, \tag{5}$$

where $J_n(.)$ is a Bessel function of the first kind in order of n and $k_1 = k_0\sqrt{\varepsilon_c\mu_c}$. Since the coated layer is an anisotropic medium, the solution of the Maxwell's wave equation in this region using the method similar to Ref. [19] and [20] takes the form as follows

$$E_z{}^{(2)} = E_0 \sum_{n=-\infty}^{+\infty} i^n \left[b_n J_{n'}\left(k_2 r\right) + c_n Y_{n'}\left(k_2 r\right)\right] e^{in\phi}, \tag{6}$$

where $Y_{n'}$ is a Bessel function of the second kind with order n' so that n' is equal to $n' = n\sqrt{\mu_t/\mu_r}$ and $k_2 = k_0\sqrt{\varepsilon_z\mu_t}$ [19].

a_n, b_n, c_n and d_n are the Lorentz–Mie coefficients and can be found using the boundary condition that is, continuity of z-component of electric field and ϕ-component of magnetic field at the two boundaries $r = R_1$ and $r = R$. If we write the boundary condition in matrix form for the boundary at $r = R$ we have

$$D_n^H(R).\begin{bmatrix}1\\ a_n\end{bmatrix} = D_{n',2}^Y(R).\begin{bmatrix}b_n\\ c_n\end{bmatrix} \tag{7}$$

and for the boundary condition at $r = R_1$ we have

$$D_{n,1}^Y\left(R_1\right).\begin{bmatrix}d_n\\ 0\end{bmatrix} = D_{n',2}^Y\left(R_1\right).\begin{bmatrix}b_n\\ c_n\end{bmatrix} \tag{8}$$

where $D_n^H(R)$ and $D_{n',m}^Y(m = 1, 2)$ are 2×2 matrix and defined as follows

$$D_n^H(R) = \begin{bmatrix} J_n(kR) & H_n^{(1)}(kR) \\ N J_n'(kR) & N H_n'^{(1)}(kR)\end{bmatrix}, \tag{9}$$

and

$$D_{n',m}^Y(R_i) = \begin{bmatrix} J_{n'}\left(k_m R_i\right) & Y_{n'}\left(k_m R_i\right) \\ N_m J_{n'}'\left(k_m R_i\right) & N_m Y_{n'}'\left(k_m R_i\right)\end{bmatrix}. \tag{10}$$

In above equations $N = \sqrt{\varepsilon_0\mu_0}$, $N_2 = \sqrt{\varepsilon_z\mu_t}$ and $N_1 = \sqrt{\varepsilon_c\mu_c}$. Finding $\begin{bmatrix}b_n\\ c_n\end{bmatrix}$ from Eq. (8) and substituting to Eq. (7) lead to

$$\begin{bmatrix}1\\ a_n\end{bmatrix} = M_n.\begin{bmatrix}d_n\\ 0\end{bmatrix},$$

where M_n is a 2×2 matrix and can be found as follows

$$M_n = \begin{bmatrix} M_{n,11} & M_{n,12} \\ M_{n,21} & M_{n,22}\end{bmatrix};$$

and

$$M_n = \left[D_n^H(R)\right]^{-1}.D_{n',2}^Y(R).\left[D_{n',2}^Y\left(R_1\right)\right]^{-1}.D_{n,1}^Y\left(R_1\right).$$

For the a_n and d_n coefficients we have

$$d_n = \frac{1}{M_{n,11}} \tag{11}$$

$$a_n = \frac{M_{n,21}}{M_{n,11}}, \tag{12}$$

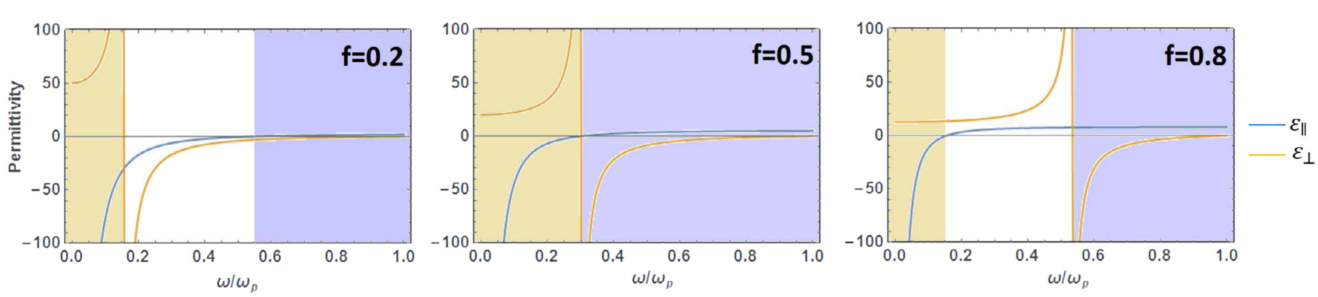

Fig. 3. Real part of the two components of effective permittivity $\epsilon_{\parallel}$(blue line) and $\epsilon_{\perp}$(orange line) in a layered metamaterial composed of a Drude medium with $\gamma = \omega_P/100$ and a dielectric layer with permittivity $\epsilon_I = 10$ for three different filling factors (a) $f = 0.2$, (b) $f = 0.5$, (c) $f = 0.8$. The shaded regions denote spectral bands where the metamaterial exhibits a hyperbolic dispersion of the type I (shaded in brown) and the type II (shaded in blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)