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Scalar and vector Hermite–Gaussian soliton in strong nonlocal media with exponential-decay response

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A R T I C L E I N F O

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A B S T R A C T

This paper investigates the propagation of scalar and vector Hermite–Gaussian (HG) solitons in strong nonlocal media with exponential-decay response. The evolution equations for the parameters of the single HG beam are obtained by variational approach and the analytical results are confirmed by numerical simulation in section 2. Both the analytical and numerical solutions show that the critical power is increase with the increase of the order. Section 3 numerically studied the vector HG soliton and found that, the total critical power and the initial powers of the components should satisfy a complex relation. Because of the mutual attraction between the components, the stability of the quasi high-order vector HG soliton is better than the corresponding single HG beam during propagation.

1. Introduction

Strong nonlocality can completely suppress the repulsion between the neighbor beam "petals" with a π phase flip of the multi-hump optical beam [1,2], thus the stable strong nonlocal multi-hump optical soliton can be formed. This characteristic of such soliton stimulates scholars with a strong interest to study it. So far, a wide variety of nonlocal multi-hump solitons have been studied, yielding a series of achievements. Such as, Hermite–Gaussian solitons [3], Laguerre–Gaussian solitons [4,5], Ince–Gaussian solitons [6], Complex-variable-function–Gaussian solitons [7], Ring dark and antidark solitons [8], necklace–ring solitons [2] and vortex solitons [9,10] et al., all of these have been proven to exist in strong nonlocal media.

However, the research described above almost assumes that the nonlocal response function has a Gaussian-shaped, which is purely academic and does not naturally appear in nonlocal physics. For instance, the response functions for lead glass [11] and $(1+1)$ dimensions nematic liquid crystal (NLC) [12], which have been verified as the strong nonlocal media, are logarithm and exponential-decay, respectively. Since the response function is not differentiable at its origin, the exact analytical higher-order soliton can hardly be achieved [13] in such media. Usually, we must resort to numerical solutions instead. For instance, Xu and Dong have numerically demonstrated the stability of multipole-mode solitons in NLC [14] and thermal nonlinear media [15], respectively. However, in order to known more about the propagation of multipole-mode solitons in NLC and thermal nonlinear media [13], it is necessary to theoretically investigate it, such as this paper obtains an approximately analytical solution of HG soliton in NLC by variational approach and confirms it by numerical simulation. Furthermore, the numerical result shows that the scalar higher-order HG soliton ($n \geq 5$) is unstable, and Ref. [1] and [2] demonstrated that the coupled propagation can enhance the stability of multipole-mode optical beams. Therefore this paper also numerically studies the HG vector soliton and found a complex relation between the total critical power and the initial powers of the components.

2. Scalar HG soliton

2.1. Theory model and variational approximation

The propagation of a paraxial optical beam in $(1+1)$-dimension nonlocal nonlinear media is modeled by the normalized nonlocal nonlinear Schrodinger equation (NNLSE) [3,16] as follows:

$$i\frac{\partial \psi}{\partial z} + \frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} + \psi \int_{-\infty}^{+\infty} R(x-x')\left|\psi(x',z)\right|^2 dx' = 0 \qquad (1)$$

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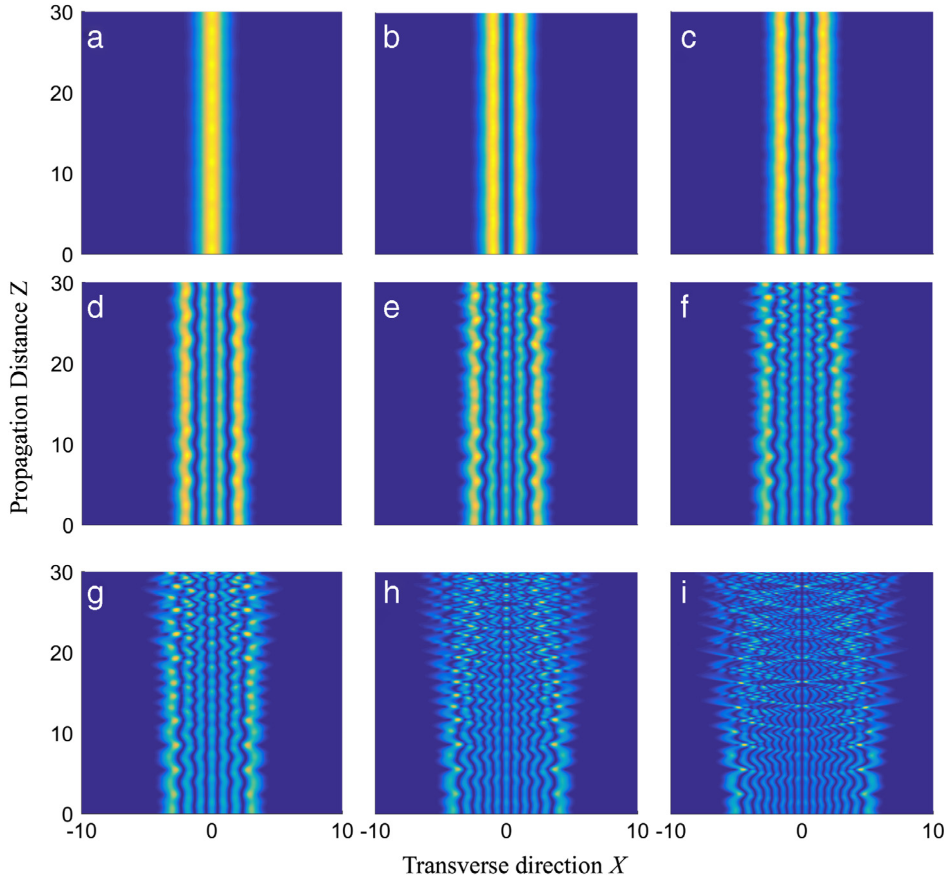

Fig. 1. Propagation of HG beam in strong nonlocal media with exponential-decay response function. The parameters are chosen as (a) $n = 0$, $P_{c0} = 1030$, (b) $n = 1$, $P_{c1} = 1840$, (c) $n = 2$, $P_{c2} = 2550$, (d) $n = 3$, $P_{c3} = 3300$, (e) $n = 4$, $P_{c4} = 3850$, (f) $n = 5$, $P_5 = 4400$, (g) $n = 6$, $P_6 = 4900$, (h) $n = 10$, $P_{10} = 6400$, (i) $n = 15$, $P_{15} = 8200$.

where $\psi(x,z)$ is the paraxial optical beam, x and z are the transverse and longitudinal coordinates which were scaled by the input beam width and diffraction length, respectively. $R(x)$ is the real normalized symmetric response function, and can be taken as exponential-decay shaped for the (1 + 1)–dimension NLC [12,13,16]

$$R(x) = \frac{1}{2\sigma}\exp\left(-\frac{|x|}{\sigma}\right) \tag{2}$$

where σ is the characteristic length of the response function.

The Lagrange density equation, which corresponding to Eq. (1), can be written as

$$L = \frac{i}{2}\left(\psi^*\frac{\partial\psi}{\partial z} - \psi\frac{\partial\psi^*}{\partial z}\right) - \frac{1}{2}\left|\frac{\partial\psi}{\partial x}\right|^2 + \frac{1}{2}|\psi|^2\int_{-\infty}^{+\infty} R(x-x')\left|\psi(x',z)\right|^2 dx'. \tag{3}$$

Assuming that the paraxial optical beam is HG shaped [3]

$$\psi(x,z) = A(z)H_n\left[\frac{x}{a(z)}\right]\exp\left[i\theta(z) + ic(z)x^2 - \frac{x^2}{2a^2(z)}\right]. \tag{4}$$

In strong nonlocal media, the response function can be expand as follows

$$R(x) = \frac{1}{2\sigma}\left(1 - \frac{|x|}{\sigma} + \frac{x^2}{2\sigma^2}\right) \tag{5}$$

where $A(z)$ represents the amplitude, $H_n[x/a(z)]$ stands for the Hermite polynomial, $\theta(z)$ is the phase of complex amplitude, $c(z)$ and $a(z)$ represent the phase-front curvature and the fundament-mode beam width, respectively. According to the definition of the second-order moment for beam width, the initial high-order HG beam widths are $(2n+1)^{1/2}a$. σ is the characteristic length of the response function. Therefore we can predict that the stability of the HG soliton is decrease as the increase of the order in general for the weakening of the nonlocality which can be expressed as $\sigma/(2n+1)^{1/2}a$.

Then the average Lagrange can be obtained by substituting Eqs. (4) and (5) into Eq. (2) and integrating Lagrange density over x

$$L = -\sqrt{\pi}2^n n! A^2\left[\frac{(2n+1)}{2}a^3\frac{dc}{dz} + a\frac{d\theta}{dz} + \frac{1}{2}(2n+1)\left(\frac{1}{2a} + 2c^2a^3\right)\right] + \frac{1}{2}\left[(2^{2n}(n!))^2\frac{A^4a^2\pi}{2\sigma} - \frac{A^4a^3b_n\sqrt{\pi}}{2\sigma^2} + (2n+1)2^{2n}(n!)^2\frac{A^4a^4\pi}{4\sigma^3}\right] \tag{6}$$

where

$$b_n = \frac{\int_{-\infty}^{+\infty} H_n^2(x)\exp(-x^2)\int_{-\infty}^{+\infty}|x-x'|\,H_n^2(x')\exp(-x'^2)dx'dx}{\sqrt{\pi}} \tag{7}$$

b_n is a constant which dependent on n, such as $b_0 = \sqrt{2}$, $b_1 = 7\sqrt{2}$, $b_2 = 145\sqrt{2}$, $b_3 = 6183\sqrt{2}$ et al.

Based on the Euler–Lagrange equations, we can obtain the evolution equations for the parameters of the optical beam

$$A^2a = A_{in}^2a_{in} = \frac{P}{2^n n!\sqrt{\pi}} \tag{8a}$$

$$\frac{da}{dz} - 2ca = 0 \tag{8b}$$

$$\frac{d\theta}{dz} = -\frac{2n+1}{2a^2} + \frac{P}{2\sigma} - \frac{3ab_nP}{8\sigma^2 2^{2n}(n!)^2} + (2n+1)\frac{a^2P}{8\sigma^3} \tag{8c}$$

$$\frac{dc}{dz} = \frac{1}{2a^4} - 2c^2 - \frac{Pb_n}{4\sqrt{\pi}2^{2n}(n!)^2(2n+1)a\sigma^2} + \frac{P}{4\sigma^3} \tag{8d}$$

where A_{in} and a_{in} are the initial amplitude and beam width, respectively. The evolution law of the beam width can be obtained by combining Eqs. (8b) and (8d),

$$\frac{d^2a}{dz^2} = \frac{1}{a^3} - \frac{b_nP}{2\sqrt{\pi}2^{2n}(n!)^2(2n+1)\sigma^2} + \frac{aP}{2\sigma^3}. \tag{9}$$