Optics Communications 403 (2017) 271–276

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Dynamics of Bell-nonlocality and entanglement in a ring cavity induced by spontaneous emission

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ARTICLE INFO

*Keywords:*
Entanglement
Bell-nonlocality
CHSH-Bell inequality
Concurrence

ABSTRACT

By considering the spatial motion of the atoms, we study the time evolution of Bell nonlocality and entanglement of a pair of atoms for two kinds of Werner-type internal states in an ideal single-mode ring cavity. We have proved that both Bell nonlocality and entanglement have the phenomena of sudden death and sudden birth for the initial state $W'_{\pm}$, while for the initial state $W_{\pm}$, Bell nonlocality has the phenomenon of sudden death, but entanglement decays to zero asymptotically over time. We also notice that the preservation of Bell-inequality violation is much shorter than that of entanglement. In addition, it is shown that the disentanglement time and the Bell-inequality violation time both depend on the purity and the width of the wave packet describing the motion of the atomic center of mass.



## 1. Introduction

As we all know, Entanglement as a kind of quantum correlation, is the characteristic feature of quantum mechanics. Because of its important applications in quantum information and quantum computation [1–4], Entanglement has under extensive research in the last few years. Due to the inevitable interaction with an environment, people find the phenomena of entanglement sudden death (ESD) [5–8] and entanglement sudden birth (ESB) [9,10] during the course of research.

Like entanglement, Bell-nonlocality is also a sort of quantum correlation beyond space, which is the unique property of quantum mechanics and cannot be reproduced by any classical local model. The presence of Bell-nonlocality is unambiguously identified by the violation of Bell inequality (especially the CHSH Bell inequality), which has been proved both theoretically and experimentally. In recent years, the relationship between entanglement and Bell-nonlocality has attracted wide attention [11–16]. Like ESD and ESB, people also find the phenomena of Bell-nonlocality sudden death (BNSD) and Bell-nonlocality sudden birth (BNSB) [17–20]. Apart from the phenomena of BNSD and BNSB, people also notice that some states with a very high value of entanglement but not showing any inherent nonlocal correlations and the only method for identification of these states is to see whether they violate the famous CHSH-Bell inequality. The Bell violations of CHSH-Bell inequality distinguish the genuine multipartite entanglement which has recently been recognized as a valuable resource that is of crucial importance in realization of many quantum computation [21,22] and quantum information, such as device-independent random number generation (DIRNG) [23] and device-independent quantum key distribution (DIQKD) [24–26].

In Ref. [27], they have studied two atoms' entanglement for two pure Bell-like states in a single-mode ring cavity. In the present paper, based on Ref. [27], we study the time evolution of Bell-nonlocality as measured by CHSH-Bell inequality for two Werner-like type states. We compare the dynamics of Bell-nonlocality with that of entanglement as measured by concurrence and conclude that the preservation of Bell-inequality violation is significantly shorter than that of the entanglement, and the disentanglement time and the Bell-nonlocality violation time are both influenced by the atomic wave packet's width. Because the two atoms are placed in an ideal single-mode ring cavity, the phenomena of collapse and revival of Bell-nonlocality and entanglement both happen.

The other parts of the present letter is arranged as follows. In Section 2, we introduce the total system's effective Hamiltonian and its eigenequation. In Section 3, we assume that initially the two atoms are prepared in the superposition state of two Gaussian wave packets and the cavity-field is prepared in a vacuum state, and derive the reduced density matrix for the two atoms' Werner-like type states by tracing off the degrees of freedom of the field and the external motion of the atoms in the relative-coordinate picture. In Section 4, the CHSH-type Bell

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http://dx.doi.org/10.1016/j.optcom.2017.07.056
Received 3 May 2017; Received in revised form 19 July 2017; Accepted 19 July 2017
0030-4018/

inequality and concurrence are briefly reviewed and purity as well as the width of the wave packet effects on the dynamics of Bell nonlocality and entanglement for two Werner-like type internal states are analyzed. In Section 5, we summarized the important conclusions of this letter.

## 2. Model and its solution

In this part, we first consider a bipartite system which is composed of two identical two-level atoms and the two atoms are affected by an ideal single-mode cavity field. A pair of atoms we considered here with the same mass $m_0$ are named as atom 1 and atom 2, separated by a transition frequency $\omega_0$ between the excited state $|e\rangle$ and the ground state $|g\rangle$. It is assumed that the two atoms are discrete in space, and the position coordinates are $\hat{x}_1$ and $\hat{x}_2$ respectively, and the corresponding momenta are $\hat{p}_1$ and $\hat{p}_2$. Thus the expressions $\hat{p} = (\hat{p}_1 - \hat{p}_2)/2$ and $\hat{P} = \hat{p}_1 + \hat{p}_2$ are the relative momentum and the center of mass (CM) of the momentum for the two atoms. Here, $\hat{X} = (\hat{x}_1 + \hat{x}_2)/2$ stands for the two atoms' CM position and $\hat{x} = \hat{x}_1 - \hat{x}_2$ is the relative position of the two atoms.

For above quantum model we considered here, whose dynamics is governed by the following Hamiltonian (here we have taken on account the condition of rotating-wave approximation):

$$\hat{H} = \frac{\hat{p}_1^2}{2m_0} + \frac{\hat{p}_2^2}{2m_0} + \frac{1}{2}\hbar\omega_0\left(\hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)}\right) + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g\left[\hat{a}(\hat{\sigma}_+^{(1)}e^{ikx_1} + \hat{\sigma}_+^{(2)}e^{ikx_2}) + H.c.\right], \quad (1)$$

where $\hat{\sigma}_z^{(i)} = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|$, $\hat{\sigma}_+^{(i)} = |e_i\rangle\langle g_i|$ and $\hat{\sigma}_-^{(i)} = |g_i\rangle\langle e_i|$ $(i = 1, 2)$ are the transition, rising and lowering operators for the $i$th atom related with the ground state $|g\rangle$ and the excited state $|e\rangle$; $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators for the cavity field, $\omega$ and $k$ are the cavity field's frequency and wave number; and $g$ is a constant which depicts the interaction of the atom and the field. The photon absorption and emission processes associated with the photon back-action to the corresponding atom can be seen from Eq. (1).

The evolution operator $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$ can be factorized into the following form:

$$\hat{U}(t) = \hat{W}(x_1)\hat{W}(x_2)\hat{U}_e(t)\hat{W}(x_2)^\dagger\hat{W}(x_1)^\dagger, \quad (2)$$

where the unitary operator $\hat{W}(x_i)$ $(i = 1, 2)$ is given by

$$\hat{W}(x_i) = \exp\left(\frac{ikx_i}{2}\right)|e_i\rangle\langle e_i| + \exp\left(\frac{-ikx_i}{2}\right)|g_i\rangle\langle g_i|, \quad (3)$$

describes the coupling of the spatial degrees with the internal levels of the atom $i$. The effective Hamiltonian $\hat{H}_e = \hat{H}_0 + \hat{H}_1$ of the system is given as follows:

$$\hat{H}_0 = \frac{\hat{p}_1^2}{2m_0} + \frac{\hat{p}_2^2}{2m_0} + \frac{\hbar^2k^2}{4m_0}, \quad (4)$$

$$\hat{H}_1 = \hbar\sum_{i=1,2}\left[\frac{\Omega_i}{2}(|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|) + g(\hat{a}^\dagger|g_i\rangle\langle e_i| + \text{H.c.})\right] + \hbar\omega\hat{a}^\dagger\hat{a}. \quad (5)$$

where the expression of the operator $\hat{U}_e(t)$ is $\hat{U}_e(t) = \exp\left(-i\hat{H}_e t/\hbar\right)$.

Here, because the velocity of a realistic atom is far smaller than the light velocity in vacuum, then we can draw the conclusion that $\Omega_1 = \omega_0 + p_1k/m_0 \approx \omega_0$ and $\Omega_2 = \omega_0 + p_2k/m_0 \approx \omega_0$. Furthermore, for simplicity, we consider the resonance case, namely, $\omega_0 = \omega$.

According to Eq. (5), the total excitation number $\hat{a}^\dagger\hat{a} + |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2|$ is conserved during the interaction and in this subspace the total excitation number is $n + 2$. By solving the eigenequation of $\hat{H}_1$, in a base formed by $|e_1, e_2, 0\rangle$, $|e_1, g_2, 1\rangle$, $|g_1, e_2, 1\rangle$ and $|g_1, g_2, 2\rangle$[27], the eigenstates of $\hat{H}_1$ are found to be of the form:

$$|\Psi\rangle_1^{(0)} = \frac{\sqrt{6}}{3}|e_1, e_2, 0\rangle - \frac{\sqrt{3}}{3}|g_1, g_2, 2\rangle, \quad (6)$$

$$|\Psi\rangle_2^{(0)} = -\frac{\sqrt{2}}{2}|e_1, g_2, 1\rangle + \frac{\sqrt{2}}{2}|g_1, e_2, 1\rangle, \quad (7)$$

$$|\Psi\rangle_3^{(0)} = \frac{\sqrt{6}}{3}|e_1, e_2, 0\rangle + \frac{1}{2}|e_1, g_2, 1\rangle + \frac{1}{2}|g_1, e_2, 1\rangle + \frac{\sqrt{3}}{3}|g_1, g_2, 2\rangle, \quad (8)$$

$$|\Psi\rangle_4^{(0)} = -\frac{\sqrt{6}}{6}|e_1, e_2, 0\rangle + \frac{1}{2}|e_1, g_2, 1\rangle + \frac{1}{2}|g_1, e_2, 1\rangle - \frac{\sqrt{3}}{3}|g_1, g_2, 2\rangle \quad (9)$$

with the eigenvalues

$$E_1^{(0)} = E_2^{(0)} = \hbar\omega_0, \qquad E_3^{(0)} = \hbar\omega_0 + \sqrt{6}\hbar g, \qquad E_4^{(0)} = \hbar\omega_0 - \sqrt{6}\hbar g. \quad (10)$$

Similarly, the total excitation number $\hat{a}^\dagger\hat{a} + |e_1\rangle\langle e_1|$ (or $\hat{a}^\dagger\hat{a} + |e_2\rangle\langle e_2|$) is conserved during the interaction and in this subspace the total excitation number is $n + 1$. By solving the eigenequation of $\hat{H}_1$, in a base formed by $|e_1, g_2, 0\rangle$, $|g_1, e_2, 0\rangle$ and $|g_1, g_2, 1\rangle$, the eigenstates of $\hat{H}_1$ are found to be of the form:

$$|\Psi\rangle_1^{(0)} = \frac{\sqrt{2}}{2}|e_1, g_2, 0\rangle - \frac{\sqrt{2}}{2}|g_1, e_2, 0\rangle, \quad (11)$$

$$|\Psi\rangle_2^{(0)} = \frac{1}{2}|e_1, g_2, 0\rangle + \frac{1}{2}|g_1, e_2, 0\rangle + \frac{\sqrt{2}}{2}|g_1, g_2, 1\rangle, \quad (12)$$

$$|\Psi\rangle_3^{(0)} = \frac{1}{2}|e_1, g_2, 0\rangle + \frac{1}{2}|g_1, e_2, 0\rangle - \frac{\sqrt{2}}{2}|g_1, g_2, 1\rangle \quad (13)$$

with the eigenvalues

$$E_1^{(0)} = 0, \qquad E_2^{(0)} = \sqrt{2}\hbar g, \qquad E_3^{(0)} = -\sqrt{2}\hbar g. \quad (14)$$

## 3. The two atoms' reduced density matrix

First of all, we may supposed that:

(i) The two atoms are in the following separable state $|\psi(0)\rangle$ at the initial time.

$$|\psi(0)\rangle = |\mu\rangle_1 \otimes |\mu\rangle_2. \quad (15)$$

Here, $|\mu\rangle_i$ $(i = 1, 2)$ denotes the $i$th atom's Gaussian wave packet describing the corresponding atom's spatial distribution, which can be expressed as:

$$\mu(x_i, 0) = \int_{-\infty}^{\infty} dp_i\, C_{p_i}\, \exp\left(\frac{i}{\hbar}p_i x_i\right) = \left(\frac{1}{2\pi d^2}\right)^{1/4}\exp\left[-\frac{(x_i + a_i)^2}{4d^2}\right], \quad (16)$$

where

$a_i$ and $d$ are the center and the width of the Gaussian function $\mu(x_i, 0)$. The coefficient $C_{p_i}$ is expressed as

$$C_{p_i} = \left(\frac{2d^2}{\pi\hbar^2}\right)^{1/4}\exp\left(-\frac{d^2p_i^2}{\hbar^2} + \frac{ia_ip_i}{\hbar}\right). \quad (17)$$

For the sake of simplicity, we suppose that $a_1 = -a_2 = a/2$. Then in the initial time, the density operator of the two atoms concerning spacial motion can be written as $\hat{\rho}_s(0) = |\psi(0)\rangle\langle\psi(0)|$.

(ii) The two atoms are in a Werner-like type state $W_\pm$ [28] specified below:

$$W_\pm = (1 - p)\frac{\Pi_4}{4} + p|\Psi_\pm\rangle\langle\Psi_\pm|. \quad (18)$$

With

$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|e_1, e_2\rangle \pm |g_1, g_2\rangle)$, $\Pi_4$ denotes the $4 \times 4$ identity matrix and the parameter $p$ indicates the purity of the initial states which ranges from 0 for maximally mixed states to 1 for pure states. Then in the initial time, the density operator of the two atoms concerning their internal state may as well be written as $\hat{\rho}_i(0) = |W_\pm\rangle\langle W_\pm|$.

(iii) The initial state of the cavity field is a vacuum state, i.e., $\hat{\rho}_f(0) = |0\rangle\langle 0|$.

Based on the above, it is easy to obtain the whole system's density operator of the initial time, which is of the form $\hat{\rho}(0) = \hat{\rho}_s(0)\,\hat{\rho}_i(0)\,\hat{\rho}_f(0)$. As time evolves, the density operator of the system becomes $\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t)$. Here, $\hat{U}(t)$ is the unitary operator in Eq. (2). In the standard basis $|e_1, e_2\rangle, |e_1, g_2\rangle, |g_1, e_2\rangle$ and $|g_1, g_2\rangle$, the elements of the