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Propagation properties of hollow sinh-Gaussian beams in quadratic-index medium

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ABSTRACT

Based on the Collins integral formula, the analytical expression for a hollow sinh-Gaussian (HsG) beam propagating through the quadratic-index medium is derived. The propagation properties of a single HsG beam and their interactions have been studied in detail with numerical examples. The results show that inhomogeneity can support self-repeating intensity distributions of HsG beams. With high-ordered beam order n, HsG beams could maintain their initial dark hollow distributions for a longer distance. In addition, interference fringes appear at the interactional region. The central intensity is a prominent peak for two in-phase beams, which is zero for two out-of phase beams. By tuning the initial beam phase shift, the distribution of the fringes can be controlled.

1. Introduction

In the past decades, dark hollow (DH) beams have attracted much attention due to their wide potential applications in atom optics. Some studies of atoms guiding, trapping and focusing with DH beams have been reported [1–7]. Conventional DH beams such as Bessel Gaussian beam [8], Laguerre Gaussian beam [9] and high-order Bessel beam [10], have been investigated widely. Y. Cai *at al* proposed a special model of DH beams named hollow Gaussian (HG) beams and showed their unique propagation properties for the first time [11,12]. It is shown that HG beams have varied beam profiles when propagating in free space, and its dark hollow beam profiles disappear in the far field, which is different from conventional DH beams. Then the propagation characteristics of HG beams through optical systems were reported in Refs. [13–15], the focusing properties of HG beams were explored in Refs. [16,17]. In addition, the beam quality of the HG beams has also been studied previously [18–20].

As an alternative model of HG beams, hollow sinh-Gaussian (HsG) beams were reported by Q. G. Sun for the first time [21]. It is demonstrated that the HsG beams can be regarded as a combination of a series of special Hypergeometric-Gaussian beams, Laguerre Gaussian beams, etc. HsG beams are characterized by single ringed transverse intensity distributions. Their intensity pattern shows a central dark hollow surrounded by a bright ring. Recently, HsG beams have been widely studied in both theoretical and experimental aspects. B. Tang *et al.* studied the propagation properties of HsG beams through fractional Fourier transform optical systems theoretically [22], X. Lu *et al.* carried out the experiment observation of the fractional Fourier transform for HsG beams [23]. Z. Zhang *et al.* reported the tight focusing properties of HsG beams [24].

Quadratic-index medium, which is one of the ideal media for the long-distance optical transmission, has been widely used to implement graded index waveguides, fibers and lenses [25–29]. However, propagation characteristics of HsG beams through the quadratic-index medium have not been reported by now to the best of our knowledge. In this paper, the analytical expression of the HsG beams in quadratic-index medium is obtained by using the Collins integral formula. Propagation characteristics of a single HsG beam is investigated numerically based on the Split-Step Beam Propagation Method (SS-BPM). In addition, the interaction characteristics of two HsG beams are discussed. The results show that inhomogeneity have profound effects on the propagation dynamics of HsG beams. They support periodical intensity distributions of HsG beams from dark hollow intensity patterns to central light spot intensity patterns in the former half of the period, and go in

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reverse in the latter. HsG beams can maintain their initial dark hollow intensity distributions for a longer distance with a higher beam order n. Furthermore, there is a long range attractive force between two HsG beams, leading to the periodical interaction properties. By tuning the phase shift, the distribution of the fringes can be controlled. This work may found potential applications in optical communications and beam interacting techniques, etc.

2. Theoretical models

2.1. Hollow sinh-Gaussian beam

The electric field of an HsG beam at $z = 0$ can be expressed as [21]:

$$E_n(r,0) = \sinh^n\left(\frac{r}{\omega_0}\right)\exp\left(-\frac{r^2}{\omega_0^2}\right), \tag{1}$$

where r is the transverse radial coordinate and n represents the order of HsG beams. If $n = 0$, Eq. (1) will be reduced to a fundamental Gaussian beam with the beam waist ω_0. If $n \geq 1$, Eq. (1) will be the mathematical model used to describe the HsG beams. HsG beams have dark hollow intensity distributions at the original plane. Not only the size of the central dark region, but also the radius of the bright ring varies with beam order n and beam waist ω_0 [17]. We can control the initial incident beam intensity distributions by changing reasonable initial n or ω_0.

2.2. Mathematical formulation

For quadratic-index medium, whose intensity dependent refractive index varies as $n(r) = n_0(1-\beta^2 r^2/2)$. n_0 is the refractive index along the spatial axis and β is the measure of the parabolic dependence of the index $n(r)$. $\beta = 0$ represents the isotropic case and $0 < \beta^2 \leqslant 1/2$ represents a medium with weakly inhomogeneity, in which the change in the refractive index over a displacement of one wavelength can be neglected.

The beam propagation in the quadratic-index medium is equivalent to the corresponding problem of Schrödinger equation for the two-dimensional harmonic oscillator [30]. Assuming that the optical field $E(r,z,t) = \psi(r,z)exp\,(i\omega t)$, one can obtain the paraxial wave equation of the slowly varying envelope $\psi(r,z)$ in the quadratic-index medium [31]:

$$2ik\frac{\partial\psi(r_\perp,z)}{\partial z} + \nabla_\perp^2\psi(r_\perp,z) - k^2\beta^2 r^2\psi(r_\perp,z) = 0, \tag{2}$$

where $\nabla_\perp^2 = \partial^2/x^2 + \partial^2/y^2$ is the two dimensional transverse Laplacian operator. Based on the ABCD optical transformation matrix methods, we could drive the mathematical formulation of HsG beams propagating in the quadratic-index medium. The transformation matrix of the quadratic-index medium can be expressed as follows [32]:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos(\beta z) & \frac{1}{\beta}\sin(\beta z) \\ -\beta\sin(\beta z) & \cos(\beta z) \end{pmatrix}, \tag{3}$$

within the framework of the paraxial approximation, HsG beams passing through the quadratic-index medium obey well known Collins integral formula [33]:

$$\begin{aligned} E_n(r',z) = &\frac{i}{\lambda B}\exp(-ikz)\times\int_0^{2\pi}\int_0^{\infty} E_n(r,0) \\ &\times\exp\left\{-\frac{ik}{2B}\left[Ar^2 - 2rr'\cos(\theta'-\theta) + Dr'^2\right]\right\} rdrd\theta, \end{aligned} \tag{4}$$

where $E_n\ (r,0)$ is the input electric fields and $E_n(r',z)$ is the output electric fields. r,θ and r',θ' are the radial and the azimuth angle coordinates in the input and output plane, respectively. The constant phase in Eq. (4), which has no influence on the output intensity distributions, could be omitted. To investigate the optical fields of HsG beams at arbitrary distance z, we can rewrite Eq. (1) as:

$$E_n(r,0) = \sum_{m=0}^{n} a_m b_m \exp\left[-\frac{(r+c_m)^2}{\omega_0^2}\right], \tag{5}$$

with coefficients given by $a_m = (-1)^m 2^{-n}\binom{n}{m}$, $b_m = \exp\left[\left(m-\frac{n}{2}\right)^2\right]$, $c_m = \omega_0\left(m-\frac{n}{2}\right)$. We can see that the nth order HsG beam can be expressed by the superposition of several decentered Gaussian beams with the same beam waist ω_0, whose center is located at the positions $(-c_m, 0)$, respectively.

Substituting Eqs. (3) and (5) into Eq. (4) and after some tedious calculation, the approximate analytical expression for the HsG beams passing through the quadratic-index medium is obtained as follows:

$$\begin{aligned} E_n(r',z) = &\frac{ik}{2B}\exp(-ikz)\exp\left(-\frac{ikDr^2}{2B}\right) \\ &\times\sum_{m=0}^{n}\sum_{s=0}^{\infty}\frac{a_m b_m(-2c_m)^s}{\omega_0^{2s}s!}\exp\left(-\frac{c_m^2}{\omega_0^2}\right)\times\Gamma\left(1+\frac{s}{2}\right), \\ &\times p^{-(1+s/2)}\times {}_1F_1\left[\left(1+\frac{s}{2};1;-\frac{q^2}{4p}\right)\right] \end{aligned} \tag{6}$$

where $p = ikA/2B + 1/\omega_0^2$, $q = kr/B$, and $J_\nu(x)$ represents ν-order Bessel function, $\Gamma(x)$ stands for the gamma function, ${}_1F_1(a;b;x)$ donates the confluent hypergeometric function. Eq. (6) provides a convenient tool for studying the propagation properties of HsG beams.

3. Simulation and Discussion

3.1. Propagation dynamics of a single HsG beam in quadratic-index medium

In this section, we investigate the propagation properties of a single HsG beam in quadratic-index medium by using the SS-BPM, in which the diffraction and inhomogeneous effects can be treated independent of each other. Influence factors of propagation distance z and beam order n are considered. Without loss of generality, we choose parameters of HsG beams as $\omega_0 = 1.0$ mm, $n = 3$. The optical wavelength λ is set to be 632.8 nm. Intensity distributions of the HsG beam at arbitrary propagating distance z have been normalized.

Fig. 1 shows the intensity distribution of a single HsG beam at different propagation z. Although quadratic-index medium cannot support HsG beams as stationary solitons, it can keep them in periodic intensity distributions. The self repeating modulation period in quadratic-index medium is given by $z_m = \pi n_0/\sqrt{n_2}$ [34], where n_2 is the nonlinear refractive coefficient. For $n_0 = 1.5$ and $n_2 = 0.01\ \mathrm{m}^{-2}$, $z_m = 47.123$ m. We select a period of $z/z_m = 0$, 0.3, 0.5, 0.7 and 1. It can be seen that in the former half of the period, the beam energy converges to the center, the on-axis intensity increase and the beam profiles of the HsG beam degenerate to a bright spot with some side lobes located sideways. In the latter half, the intensity evolution is an inverse process of the former period. At $z = z_m$, the intensity distribution is the same as the initial state exactly. This interesting propagation phenomenon can be explained by the fact that the HsG beams are not a pure mode, but a superposition of several modes. Different modes overlap and interfere in the propagating process, leading to the special properties of the HsG beams. It is identical with the beam propagation dynamics in the strongly nonlocal nonlinear (SNN) medium, in which the characteristic length of the material response function is much larger than the beam width. For SNN medium, the nonlocal nonlinear Schrödinger equation can be deduced to the simple Snyder–Mitchell model. As reported in the Ref. [35], when the input power P_0 is choose properly, the self-induced waveguide related to the input power P_0 can be generated. In this case, beam propagation through the self-induced waveguide can be regarded as the similar process in quadratic-index medium, producing the periodical self-repeating phenomenon. Moreover, it is reported that nonlocal nonlinear response of optical medium allows for the existence of stable elliptically shaped modulated self-trapped rotating singular optical beams, azimuthons, and stable rotating dipole beams [36–39]. The reports may be helpful to investigate the asymmetric properties of HsG beams, which is quite different from the diverse properties in local quadratic-index medium.

To explore the effects of different values of n on the HsG beams in quadratic-index medium, the intensity distribution of HsG beam at