Physica E 86 (2017) 117–123

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Phase transitions in normal mode spectra of two-dimensional clusters in an anisotropic power-law confining potential

Assia Bessaa*, Mourad Djebli

Theoretical Physics Laboratory, Faculty of Physics U.S.T.H.B., P.B.32 Bab-Ezzouar, 16079 Algiers, Algeria

ARTICLE INFO

Keywords:
Mesoscopic 2D systems
Phase transitions
Clusters

ABSTRACT

We present a numerical analysis of several phase transitions which take place in the eigenmode spectrum of a two-dimensional (2D) logarithmic cluster subjected to an anisotropic power law confinement. Varying the anisotropy in a non-parabolic soft confinement drives the system to undergo structural phase transitions of first order, while for a hard wall confinement this variation affects strongly the eigenmode spectrum and breaks the symmetry of the system due to the removal of degeneracy and the coupling between some normal modes.

1. Introduction

Two-dimensional finite clusters have attracted much interest over the past few decades due to their occurrence in a wide range of real systems like electrons in quantum dots [1] or on the surface of liquid Helium [2], charged particles on colloidal suspensions [3] and dusty plasmas [4]. Theoretical works based on Monte Carlo and Molecular Dynamics simulations [5–9] have revealed that the main feature of such 2D systems is that particles can arrange themselves into ring-like structures due to competing effects induced by the inter-particle interaction and the confinement in which the system is trapped. Depending on the possible charging processes, particles size and in general on the experimental conditions, the interaction between the particles may be different from one system to another, ranging from short-range to long-range interactions. Consequently, the resulting cluster undertakes a special dynamic which leads to different phase transitions with respect to some order parameters.

Other interesting real systems are charged metallic balls [10] and 2D vortex clusters which occur in mesoscopic superconductors or superfluids [11,12]. These clusters with a logarithmic inter-particle interaction [13,14] may be considered as electrons in artificial atoms that show self-organized patterns for a small number of electrons. For general 2D clusters, detailed investigations of the ground state and normal modes properties have been done in Refs. [15–19]. Previously, it was found [20,21], that decreasing the anisotropy parameter from $1 \to 0$ leads to a sequence of first and second order phase transitions for a system in a parabolic trap. This effect was studied earlier in Ref. [22] where confined ion clusters are found also to exhibit structural phase transitions with respect to the anisotropy of the confinement. The critical anisotropy at which phase transitions take place is found to be proportional to a power of the number of confined ions. For this latter cluster, an experimental observation of the zig-zag transition and two more complicated transitions to a 2D configuration are identified recently by Yan et al. [23]. In Ref. [24] both effects of shielding and anisotropy have been reported on the structure of 2D and 3D clusters. Additionally, the anisotropy can be used to destabilize vortex clusters in Bose-Einstein condensates, which are more stable in the isotropic limit [25]. Recently, Laut et al. [26] showed that the anisotropy can enhance a mode-coupling instability in plasma crystals [27]. Almost all the previous analysis have been realized within the parabolic confinement potential. However, the presence of anharmonicity in real systems may conduct to a new behavior in the structure and the spectrum of 2D clusters [28]. Within Ginzburg-Landau theory, it was found [29] that the analytic form of the confinement has an eminent role in the occurrence of zig-zag phase transitions.

In the present paper, we extend the previous investigations made in an anisotropic parabolic confinement, to a general power-law anisotropic trap where we consider 2D logarithmic clusters. Working on a 2D system is motivated by the fact that power-law traps are more easily produced in 2D than in 3D space [30]. For 2D systems a variety of experimental setups exists for producing power-law confinements ($n > 2$) and even a hard wall confinement [30–32]. We study the combined effect induced by the variation of both of anisotropy and confinement power on the structure and normal modes of 2D clusters. Phase transitions are classified with respect to jumps, softening and coupling in the eigenmodes spectrum. Furthermore, the variation of the highest frequency normal mode with the anisotropy parameter is determined in different confinements, and a special attention is paid to the behavior of some intermediate frequencies in the hard wall trap.

The paper is outlined as follows: In Section 2 we present our model

* Corresponding author.
E-mail addresses: phys_maths1@yahoo.fr (A. Bessaa), mdjebli@usthb.dz (M. Djebli).

http://dx.doi.org/10.1016/j.physe.2016.10.018
Received 21 April 2016; Accepted 11 October 2016
Available online 13 October 2016

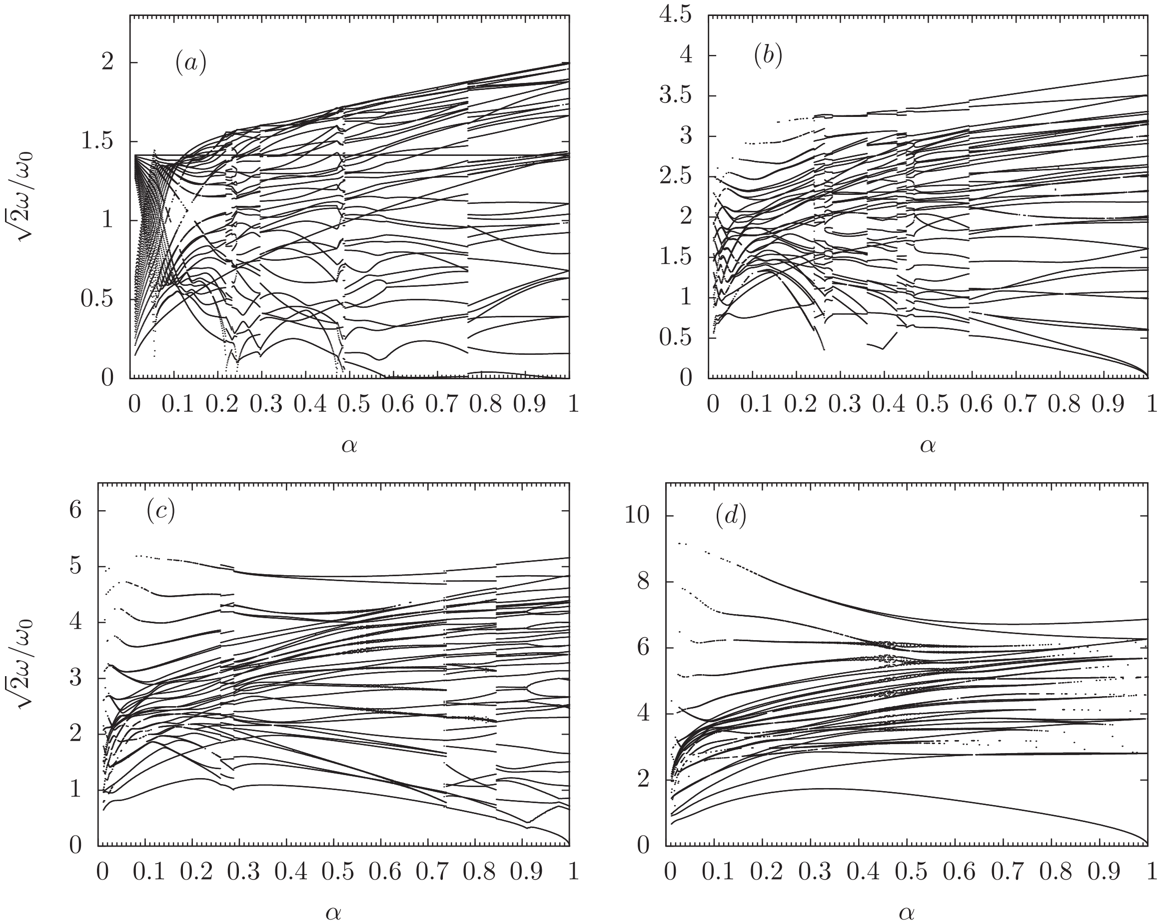

Fig. 1. the dependence of the eigenfrequency spectrum on the anisotropy parameter α for different confinement powers: $(a) n = 2$, $(b) n = 3$, $(c) n = 4$ and $(d) n = 6$ for a cluster with N=19 particles.

system. In Section 3, the behavior of the eigenmodes is studied in both soft and hard anisotropic confinements. Section 4 is devoted to main conclusions.

2. Numerical model

We consider a two-dimensional (2D) cluster of N equally mesoscopic charged particles trapped by an external power-law anisotropic confinement and interacting through a logarithmic type interaction which is a solution of the Poisson equation in 2D. Thus, our system is described by the Hamiltonian,

$$H = \frac{1}{2}m\omega_0^2 R^2 \sum_{i=1}^{N}\left(\frac{\alpha x_i^2 + y_i^2}{R^2}\right)^{n/2} - \beta\sum_{i>j}^{N}\ln\left(\frac{|\mathbf{r}_i - \mathbf{r}_j|}{R}\right), \tag{1}$$

where m is the mass of the particles, ω_0 is the radial confinement frequency, n and α are respectively, the confinement power and the anisotropy parameter. R is the length of the major axis in the limit of the hard wall, i.e, $n \to \infty$, $\mathbf{r}_i(x_i, y_i)$ is the vector coordinates of the ith particle and β is the particle-coupling constant.

We can write the Hamiltonian in Eq. (1) in dimensionless form if we express the coordinates and the energy in the following units: $r_0 = \beta^{1/n}\gamma^{-1/n}R^{(n-2)/n}$, $E_0 = \beta$, $\gamma = \frac{1}{2}m\omega_0^2$. The Hamiltonian in a dimensionless form is given by:

$$H = \sum_{i=1}^{N}(\alpha x_i^2 + y_i^2)^{n/2} - \sum_{i>j}^{N}\ln|\mathbf{r}_i - \mathbf{r}_j| \tag{2}$$

The total energy of the system involves the following parameters: particle number N, confinement power n and the anisotropy parameter α: $0 \to 1$. All the results are given in dimensionless units. To obtain the minimum energy configuration (global minimum), we used the Hamiltonian in Eq. (2) to minimize the energy of the system. For this end, we followed the Monte Carlo simulations technique firstly used in Ref. [33]. To be confident that we have found the ground state configurations, we run the MC subprogram many times starting with different random initial configurations. The MC method is extended by the Newton optimization in order to enhance the accuracy of the energy and accelerate the convergence towards its minimum value for systems which have also metastable states.

Once the ground state configuration of the system is calculated, the normal modes are obtained from the dynamical matrix A defined as:

$$A = \begin{pmatrix} \frac{\partial^2 H}{\partial x_i \partial x_j} & \frac{\partial^2 H}{\partial x_i \partial y_j} \\ \frac{\partial^2 H}{\partial y_i \partial x_j} & \frac{\partial^2 H}{\partial y_i \partial y_j} \end{pmatrix}.$$

The elements of A consist of second derivative of the energy given by Eq. (2) with respect to particles coordinates x_i,y_i. The eigenvalues and the eigenvectors of the dynamical matrix A are calculated with an accuracy of 10^{-9} using QR-Hessenberg algorithm [34]. Actually, for a system with N particles we obtain exactly $2N$ modes as a solution of the following linear system:

$$A\vec{\chi} = \lambda\vec{\chi} = \omega^2\vec{\chi} \tag{3}$$

$\vec{\chi} = (x_1, \ldots x_N, y_1, \ldots y_N)^T$ and ω^2 are eigenvectors and squared eigenfrequencies of the system, respectively.