



## Two-port two impedances fractional order oscillators



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### ABSTRACT

This paper presents a study for general fractional order oscillator based on two port network where two topologies of oscillator structure with two impedances are discussed. The two impedances are chosen to be fractional elements which give four combinations for each topology. The general oscillation frequency, condition and the phase difference between the two oscillatory outputs are deduced in terms of the transmission matrix parameter of a general two port network. As a case study: two different networks are presented which are op-amp based circuit and non-ideal gyrator circuit. The oscillation parameters for each case have been derived, and discussed numerically using Matlab. Spice simulations are presented for some cases to validate the proposed idea. Experimental results for the op-amp network are introduced to validate the reliability of the presented oscillator. The extra degree of freedom provided by the fractional order parameter enables the oscillation frequency band to cover from small Hz to hundreds MHz which is suitable range for most of measuring applications.

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### 1. Introduction

The fractional calculus has been known since the conventional calculus for centuries ago. It is a generalization of integration and differentiation to non-integer order fundamental operator. The basic definition of the Riemann–Liouville (RL) notation of fractional integral of order  $\alpha > 0$  is given by [1,2]:

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad (1)$$

where  $J^\alpha$  represents the fractional integral operator of order  $\alpha \in \mathbb{R}^+$ ,  $f(t)$  is a causal function and  $\Gamma(\cdot)$  is the gamma function. The most frequently used definitions for the general fractional order derivatives are the RL and the Caputo definitions [1,2] which represented respectively by:

$$D_{t_0}^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^t f(u)(t-u)^{m-\alpha-1} du = D^m J^{m-\alpha} f(t), \quad (2a)$$

$$D_{t_0}^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^t f^{(m)}(u)(t-u)^{m-\alpha-1} du = J^{m-\alpha} D^m f(t), \quad (2b)$$

where  $m$  is an integer such that  $(m-1) < \alpha < m$ . A very interesting property of the RL definition is that the fractional derivative of a constant is not equal to zero as by Caputo definition. Despite having lots of definitions for fractional order operators, yet it was far from application point of view due to its complexity and absence of physical and geometric interpretation. That was changed when Podlubny proposed a convincing explanation of the fractional phenomena [1]. He introduced both a geometric interpretation of the RL fractional integral as well as a physical interpretation for the RL and the Caputo fractional differentiation. Also, many numerical techniques towards the computation of fractional calculus were presented which facilitates its use in the area of science and engineering [2–16]. Fractional calculus made a breakthrough in control engineering [5,6], signal processing [7], stability analysis [8] and electrical engineering [9–18].

The Laplace transform of RL definition described by:

$$\mathcal{L}\{D_0^\alpha f(t)\} = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k D_0^{\alpha-k-1} f(t) \Big|_{t=0}, \quad (3a)$$

while Laplace transform of Caputo is as follows:

$$\mathcal{L}\{D_0^\alpha f(t)\} = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \quad (3b)$$

where  $n$  is the integer such that  $(n-1) < \alpha < n$ . Either of them is chosen based on which type of initial condition required. The

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Caputo definition allows the use of initial conditions based on integer derivatives with known physical interpretation. Under zero initial conditions, the two definitions are equivalent and described by:

$$\mathcal{L}\{D_0^\alpha f(t)\} = s^\alpha F(s). \quad (4)$$

From (4), the concept of fractance device arises. It is considered a challenge for many researchers to transform the theoretical basis to a feasible device. Although it is not yet commercially available; there are promising researches towards its realization which give a leap to its market availability soon [12–16]. The fractional-order element is considered as a general element where the conventional elements such as resistor, capacitor, and inductor are considered special cases corresponding to  $\alpha = \{0, -1, 1\}$  respectively.

The two port network concept is a very valuable technique in circuit theory and analysis. Fig. 1(a) depicts a general two port network which can be described by its transmission matrix parameter as follows:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [A] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}. \quad (5)$$

It is suitable for analyzing cascaded networks. The two port network concept adds simplicity of analysis without getting through internal details of the used device as if it is a black box. It has been used in many engineering applications [19–21]. In [19], the two port network concept was used in building transformation to LC ladder filters using current feedback operational amplifier as an active building block. A simplified amplifier analysis technique using two-port transmission parameters was derived in [20]. The derived expressions are valid for both BJT and MOS-based amplifiers and are independent of any particular small signal transistor model. With the use of two port network, a classification of Colpitts oscillators into three categories based on the specific active device terminal which is grounded was presented in [21]. General and accurate characteristic equations independent of any particular transistor (or active device) model were derived for all three

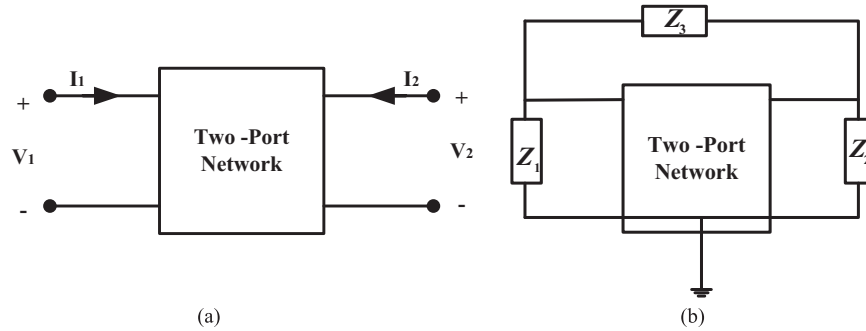


Fig. 1. (a) Two port network block, (b) Two port network oscillator with three impedance.

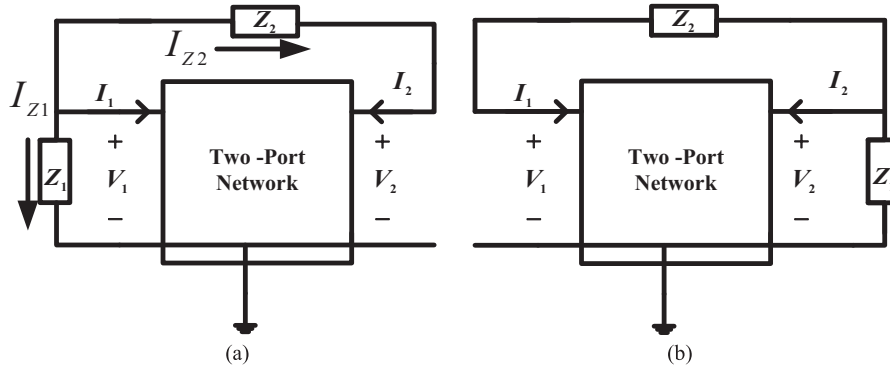


Fig. 2. Oscillators based on two-port network (a) topology 1, and (b) topology 2.

Table 1

General characteristic equation for topology 1.

Impedances combinations			Characteristic equation
#	$Z_1$	$Z_2$	
1	$\frac{1}{s^\alpha C_1}$	$\frac{1}{s^\beta C_2}$	$s^{\alpha+\beta} + \frac{((1-a_{11})(a_{22}-1)+a_{12}a_{21})}{a_{12}C_1} s^\beta + \frac{a_{11}}{a_{12}C_2} s^\alpha + \frac{a_{21}}{a_{12}C_1C_2} = 0$
2	$\frac{1}{s^\alpha C_1}$	$s^\beta L_1$	$s^{\alpha+\beta} + \frac{a_{21}L_1}{a_{11}C_1} s^\beta + \frac{a_{12}C_1}{a_{11}C_1L_1} s^\alpha + \frac{((1-a_{11})(a_{22}-1)+a_{12}a_{21})}{a_{11}C_1L_1} = 0$
3	$s^\alpha L_1$	$\frac{1}{s^\beta C_1}$	$s^{\alpha+\beta} + \frac{a_{12}C_1}{((1-a_{11})(a_{22}-1)+a_{12}a_{21})C_1L_1} s^\beta + \frac{a_{21}L_1}{((1-a_{11})(a_{22}-1)+a_{12}a_{21})C_1L_1} s^\alpha + \frac{a_{11}}{((1-a_{11})(a_{22}-1)+a_{12}a_{21})C_1L_1} = 0$
4	$s^\alpha L_1$	$s^\beta L_2$	$s^{\alpha+\beta} + \frac{a_{11}L_2}{a_{21}L_1L_2} s^\beta + \frac{((1-a_{11})(a_{22}-1)+a_{12}a_{21})L_1}{a_{21}L_1L_2} s^\alpha + \frac{a_{12}}{a_{21}L_1L_2} = 0$

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