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Emulation of current excited fractional-order capacitors and inductors using OTA topologies

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ABSTRACT

A novel topology suitable for emulating fractional-order capacitors and inductors using current excitation is achieved using a fractional-order differentiator/integrator block and appropriately configured Operational Transconductance Amplifiers. The scheme is capable of emulating both fractional-order capacitors and fractional-order inductors without any modifications to its structure. This implementation allows for electronic tuning of the order, capacitance/inductance, and bandwidth of operation by modification of only the bias current. Post-layout simulation results of the impedance magnitude and phase confirm the correct emulated behavior of both fractional-order capacitors and inductors. Two examples highlight the applications of this topology in i) realizing a fractional-order bandpass filter and ii) emulating a Cole-impedance model for biological applications. For both examples the characteristics of each circuit are validated in simulation.

1. Introduction

Fractional-order capacitors (FC), also commonly referred to as constant phase elements (CPEs), and fractional-order inductors (FI) are very important elements for implementing fractional-order systems [1]. These systems have been used to implement analog filter circuits with precise control of the attenuation characteristics [2–4], oscillator circuits with high oscillation frequencies independent of capacitance [5,6], and control systems with improved performance [7]. Unfortunately, these elements are not yet commercially available but significant research effort is ongoing to develop FCs as stand-alone two-terminal devices. For example, FCs have been developed utilizing electrolytic processes [8], fractal structures on silicon [9], dipping capacitive type polymer-coated probes in a polarizable medium [10,11] and most recently using graphene [12]. Each of the aforementioned solutions is not commercially available and also does not offer any possibilities of on-the-fly adjustability. Typical techniques for emulating a FC rely on passive *RC* trees with component values obtained using one of several suitable methods (such as Continued Fraction Expansion) [13–17]. Each technique attempts to emulate the FC impedance given by

$$Z_{FC}(s) = 1/\hat{C}s^{\alpha} \tag{1}$$

where $(0 < \alpha < 1)$ is the order and $\hat{C}$ is the pseudo-capacitance expressed in F s$^{\alpha-1}$, and $s^{\alpha} = (j\omega)^{\alpha} = \omega^{\alpha}[\cos(\alpha\pi/2) + j\ \sin(\alpha\pi/2)]$. While these approximations are successful at emulating the impedance, they are problematic when it is desired to change the characteristics $(\hat{C}, \alpha)$ of the emulated FC; requiring each of the passive components to be changed to a new value. In other words, only a fixed approximation for a specific $(\hat{C}, \alpha)$ valid over a certain pre-specified bandwidth with acceptable magnitude and phase errors is possible with these techniques. Recent works have explored active topologies using Current Feedback Operational Amplifiers (CFOAs), passive resistors, and capacitors to emulate FCs and FIs [18]. While this scheme is effective at emulating the FC and FI characteristics without any change in its structure, it suffers from the absence of any electronic tuning capabilities. An alternative solution was introduced in [19], where the employed active elements were Operational Transconductance Amplifiers (OTAs). As a result, in addition to emulating FCs and FIs without any changes in the topology, the proposed scheme allowed electronic tuning of the order, capacitance/inductance, and bandwidth of operation. However, this solution is only capable of emulating a voltage excited FC, and not a current excited FC. Here, we present a method to emulate current excited FCs and FIs using an OTA

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topology. The work is organized as follows: the proposed scheme is presented in Section 2 and its behavior is evaluated in Section 3, using the Analog Design Environment of the Cadence software and the Design Kit provided by the AMS 0.35 μm CMOS process. Two design examples are presented to highlight the applications of this topology, (i) realizing a bandpass filter through a component substitution of the corresponding passive prototype and (ii) emulating the Cole-impedance model for biological applications.

2. Proposed emulation scheme

In the previous section, the impedance of a FC is given by (1). From which the value of the capacitance (C) of a FC, in Farads, is calculated using

$$C = \hat{C}/\omega^{1-\alpha} \tag{2}$$

Highlighting that the capacitance of a FC is dependent on both frequency and order, with phase (θ_{FC}) of the impedance equal to $-\alpha\pi/2$. Similarly, the impedance of a FI is expressed by

$$Z_{FI}(s) = \hat{L}s^{\alpha} \tag{3}$$

With the relationship between the pseudo-inductance ($\hat{L}$), with units H s$^{\alpha-1}$, and conventional inductance (L) in Henries given by

$$L = \hat{L}/\omega^{1-\alpha} \tag{4}$$

with phase (θ_{FI}) of the impedance equal to $+\alpha\pi/2$.

While the emulation of voltage-excited FC and FI impedances was previously presented in [19], this topology cannot emulate the current-excited behavior of the fractional-order impedances. However, the current-excited behavior can be emulated using the topology given in Fig. 1(a). This topology is constructed using a voltage-mode block, denoted by transfer function $H(s)$, which implements a fractional-order differentiator/integrator with unity-gain frequency $\omega_0 = 1/\tau$, and appropriately configured OTAs with small-signal transconductance equal to g_m. Note that the leftmost OTA in Fig. 1(a) performs the implementation of a low impedance node required for admitting the input current. Assuming that the impedance of the differentiator/integrator block is infinite, then $i = i_1 + i_2 - i_3$. Therefore, the equivalent impedance ($Z_{eq} \equiv v/i$) is given by the expression

$$Z_{eq} = \frac{1}{g_m H(s)} \tag{5}$$

Taking into account that $H(s)$ could be equal to $(\tau s)^{\alpha}$ or $1/(\tau s)^{\alpha}$, and using (1), (3) and (5), it is obtained that

$$\hat{C} = g_m \tau^{\alpha} \tag{6}$$

$$\hat{L} = \tau^{\alpha}/g_m \tag{7}$$

Next, using (2) and (4), the expressions for (6) and (7) can be written as:

$$C = \frac{g_m}{\omega_0^{\alpha}\omega^{1-\alpha}} \tag{8}$$

$$L = \frac{1}{g_m \omega_0^{\alpha}\omega^{1-\alpha}} \tag{9}$$

The corresponding scheme for the emulation of a floating current excited FC/FI is given in Fig. 1(b). The emulated impedance, $Z_{eq} \equiv (v_1 - v_2)/i$, is also given by (5) and the relations (6)–(9) are still valid.

The two-input fractional-order differentiator/integrator block (denoted by $H(s)$) in Fig. 1(b) is realized using the topology in Fig. 2 [19]. The details of this realization are repeated here for completeness. The transfer function of the ideal fractional-order differentiator and a second order approximation of it as a function of its order (α) [20,21] are given by:

$$H(s) = (\tau s)^{\alpha} \tag{10}$$

$$H(s) \cong \frac{(\alpha^2 + 3\alpha + 2)\tau^2 s^2 + (8 - 2\alpha^2)\tau s + (\alpha^2 - 3\alpha + 2)}{(\alpha^2 - 3\alpha + 2)\tau^2 s^2 + (8 - 2\alpha^2)\tau s + (\alpha^2 + 3\alpha + 2)} \tag{11}$$

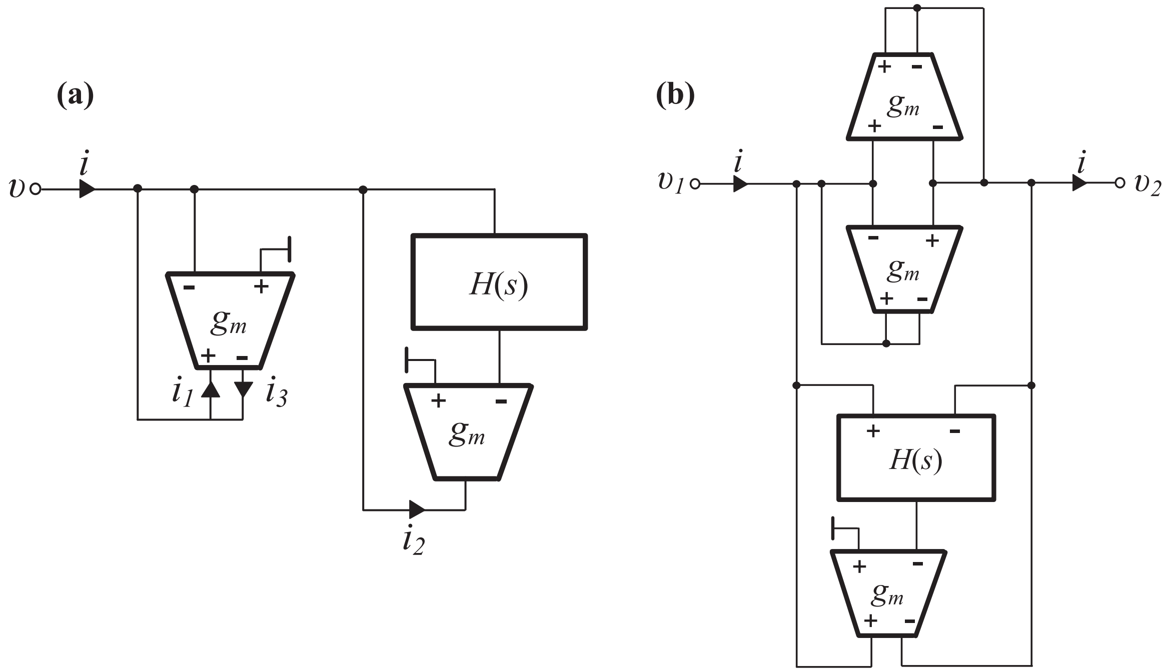

Fig. 1. Proposed emulation schemes for the current excited (a) grounded and (b) floating fractional-order capacitor and fractional-order inductor.