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Induced resonance evolution of the channeling electron beam

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ABSTRACT

The motion of channeling particles in the accompanying coordinate system can be considered as one-dimensional oscillator (in the case of planar channeling) or as a two-dimensional atom (in the case of axial channeling) (Lindhard, 1965; Baryshevskii, 1982; Vorobiev, 1984; Kalashnikov, 1988; Ryabov, 1994). The transversal motion of the channeling particles is characterized by discrete spectrum. The occupation probability of the transversal motion levels depends on the entrance angle of charged particle relative to the crystallographic axis. Passing through the single crystal the charged channeling particle undergoes periodic impact of the lattice atoms (Kalashnikov, 1988; Foot, 2005; Ter-Mikaelyan, 1972; Okorokov et al., 1973) with the main frequency $\omega = 2\pi\frac{v}{d}\gamma$, where d is the lattice constant, v and $\gamma = \frac{E}{mc^2}$ are the speed and the Lorentz-factor of the channeling particles (Kalashnikov, 1988). If an external periodic action frequency coincides with the transition frequency of the channeling particles from one quantized state of transversal motion in another, then the resonant excitation of channeling particles is possible. This effect is analogous with the excitation of the atomic electrons by the periodic field of monochromatic electromagnetic waves (Ter-Mikaelyan, 1972; Okorokov et al., 1973; Ohtsuki, 1983). The resonance conditions are discussed and the dependence of the occupation probability of the transversal motion levels of the channeling particles (electrons) upon the single crystal thickness is analyzed.

1. Introduction

A channeling particle in the accompanying coordinate system (ACS) (i.e. in the coordinate system which moves with the longitudinal velocity of channeling particles, $v = p_z/E$, $\beta = V/c = p_z/Ec$) can be treated as a one-dimensional oscillator (in the case of planar channeling), or as a two-dimensional atom (in the case of axial channeling [1–5]). From a quantum point of view the channeling particles are characterized by the energy discrete spectrum of the transversal motion [2–5]. The Doppler-effect [2,4] affects the energy levels (zones) system significantly. This suggests that for the channeling particles the well-known effects in atomic physics may be taken place, particularly when interacting with the electromagnetic field, it is possible to observe one of the well-known effect of an oscillation of an occupation of two-levels system (Rabi oscillations [6]).

Passing through the single crystal a charged particle experiences the periodic impact atoms lattice ($T = d/v$, where d is the lattice constant and v is the particle velocity). In [4,7–9] it was shown that the crystallographic lattice, acting to charged particles into a single crystal, at high energies can be represented as a superposition of a monochromatic waves with frequencies

$$\omega_n = \frac{2\pi}{d}\frac{vn}{\sqrt{1-\frac{v^2}{c^2}}} = 2\pi\frac{v}{d}\gamma n, \tag{1}$$

where $n = 1, 2, \ldots$ is the harmonics number and $\gamma = E/mc^2$ is the Lorentz factor of the channeling particle [4]. The channeling particle from the quantum point of view is characterized by discrete spectrum of transversal motion [2–5]. If an external periodic frequency of exposure by the crystal lattice (1) coincides with the transition frequency of the moving channeling particles from one quantized state to another, the resonant excitation of channeling particles may take place, similar to the atomic electron excitations by the periodic field of monochromatic electromagnetic waves [6–11].

Therefore, the resonance transition condition between levels of the transversal motion of the channeling particle can be written as:

$$\hbar\omega_n = \hbar\omega_f - \hbar\omega_i = \Delta_{fi} = \hbar\omega_{fi}. \tag{2}$$

First Okorokov [8] indicated the possibility of the coherent excitation of ions, moving along the crystallographic channels by the periodic field of single crystal.

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2. The choice the transversal motion potential

If the charged particle enters into single crystal at an entrance angle respect to crystallographic axis less than the Lindhard angle $\theta_L \approx \sqrt{2U/E}$, then it starts to move in channeling regime [1]. This means that the particle flies quite long distances within the channels formed by the crystallographic axes. The motion across these channels is limited to distances of the order of the lattice constant. From the quantum mechanics point of view, the transversal motion will be characterized by a discrete set of energy levels. The basic idea, greatly simplifies the theoretical description of the channeling effect, is to replace the true potential by the average continuous potential [4].

The periodic lattice potential is written in the form:

$$V(\mathbf{R}) = \sum_{\mathbf{g}} V_{\mathbf{g}} \exp(-i\mathbf{g}\mathbf{R}), \tag{3}$$

where $\mathbf{g}$ is the reciprocal lattice vector; $\mathbf{R}$ is the radius-vector of the channeling particle. To determine $\mathbf{R}$ we use an expression $\mathbf{R} = \mathbf{v}t + \mathbf{r}$, where $\mathbf{r}$ is the radius-vector of the channeling particle (in the accompanying coordinate system moving with longitudinal velocity of the channeling particle $\mathbf{v}$). By using the expressions for a transverse and longitudinal components of the reciprocal lattice vector as $\mathbf{g}$ and g_z ($g_z = l/d$, $l = 0, 1, 2\ldots$), we get

$$\begin{aligned} V(\mathbf{R}) &= \sum_{\mathbf{g}} \sum_{g_z} V_{\mathbf{g},g_z} \exp(-2\pi i\mathbf{g}\mathbf{r} - 2\pi i g_z v t) \\ &= i\hbar \sum_{l} F^l \exp(-2\pi i g_z v t), \end{aligned} \tag{4}$$

where

$$F^l = \frac{1}{i\hbar} \sum_{\mathbf{g}} V_{\mathbf{g},2\pi l/d} \exp(-2\pi i\mathbf{g}\mathbf{r} - 2\pi i l z/d), \quad l = 0, 1, 2, \ldots \tag{5}$$

The first term with $l = 0$ in (5) corresponds to the average continuous potential, which ensures the channeling motion [1,4].

We choose the potential well Pöschl-Teller as the averaged continuous planar potential that looks [11]

$$U(x) = -\frac{U_0}{\cosh^2\left(\frac{x}{b}\right)}, \tag{6}$$

where the parameters b and U_0 are fitted from the best approximation of the model potential to the exact one. For example, when the channeling electrons moving along the (110) plane of the tungsten single crystal in paper [14] the following values were used $b = 0.275$ Å, $U_0 = 132.8$ eV. In such potential (6), there are two families of the Schrödinger equation solutions: one is even and other is odd. These solutions are well known [11,13] and can be written as

$$\Psi_n(x) = \frac{C_n}{\cosh^{\alpha_n}\left(\frac{x}{b}\right)} P_n^{(\alpha_n,\alpha_n)}\left(\tanh\left(\frac{x}{b}\right)\right), \tag{7}$$

where $P_n(\xi)$ is the Jacobi polynomials, $\alpha_n = -\frac{1}{2}\left(1 + 2n - \sqrt{1 + \frac{8mU_0b^2}{\hbar^2}}\right)$, $n = 0, 1, 2, \ldots$, and the coefficients C_n are determined by the normalization condition of the wave functions $\int_{-\infty}^{\infty} \Psi_n^2 dx = 1$. The expression for determining the discrete energy levels is writing in the form

$$E_n = -\frac{\hbar^2}{8mb^2}\left[-(1+2n) + \sqrt{1 + \frac{8mU_0b^2}{\hbar^2}}\right]^2. \tag{8}$$

The number of the energy levels is the finite quantity and is determined by the condition $n < \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8mU_0b^2}{\hbar^2}}\right)$. In this case there are two energy levels - one even and one odd. Furthermore

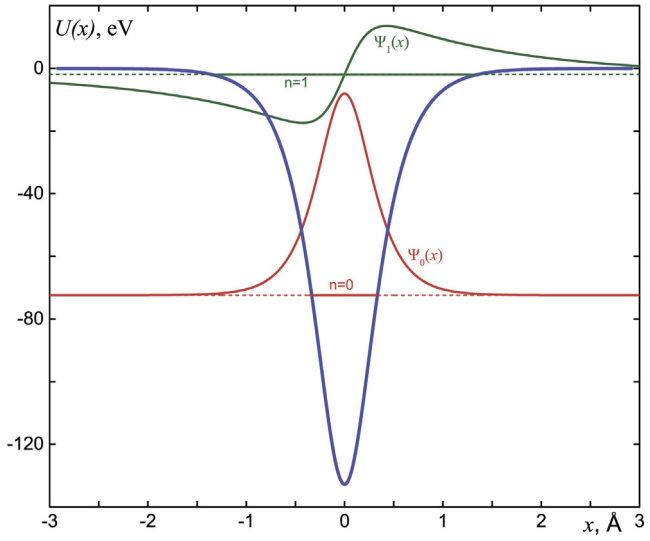

Fig. 1. Energy levels in the potential well (6).

we will examine all processes in the dipole approximation, in which matrix element $\propto \langle\Psi_i|x|\Psi_k\rangle$, involves the integration along the entire direct $x \in (-\infty; \infty)$, where integration is maintained along the entire direct and therefore the allowed transitions are possible only between states with the different parity.

The energy levels have the following values (Fig. 1):

$$E_0^+ \approx -72.39 \text{ eV}, \quad E_1^- \approx -1.98 \text{ eV} \tag{9}$$

By using the Rodrigues' formula for the Jacobi polynomials [12]

$$P_n^{(\beta,\gamma)}(\xi) = \frac{(-1)^n}{2^n n!} \frac{1}{(1-\xi)^\beta (1+\xi)^\gamma} \frac{d^n}{dx^n}\left((1-\xi)^{\beta+n}(1+\xi)^{\gamma+n}\right), \tag{10}$$

and calculating the normalization integrals, we have finally obtained the wave functions for the bound states

$$\begin{aligned} \Psi_0(x) &= \frac{1}{\sqrt{4^{(\alpha_0-1/2)}\mathrm{B}(\alpha_0,\alpha_0)b}} \frac{1}{\cosh^{\alpha_0}(x/b)}, \\ \Psi_1(x) &= \sqrt{\frac{2\Gamma(\alpha_1+3/2)}{\sqrt{\pi}\Gamma(\alpha_1)b}} \frac{\tanh(x/b)}{\cosh^{\alpha_1}(x/b)} \end{aligned} \tag{11}$$

where $\mathrm{B}(\varepsilon, \xi)$ is the beta-function and $\Gamma(\zeta)$ is the gamma-function [2].

3. The crystalline lattice atoms as a periodic perturbation

The wave function of the incident electron is represented as the plane wave

$$\Psi(x) = \exp\left(\frac{ip_\perp x}{\hbar}\right), \tag{12}$$

where $p_\perp \approx p\theta \approx p_z\theta$, ($p_z$ and $p_\perp$ are the longitudinal and transversal components of the particle momentum [4]).

By using the continuity condition of the electron wave function we obtain the occupation coefficients of the energy levels of a transversal motion of the channeling electron, which are the squares modules of the decomposition coefficients:

$$\exp\left(\frac{ip_\perp x}{\hbar}\right) = \sum_n Q_n(p_\perp)\Psi_n(x), \tag{13}$$

where $Q_n(p_\perp)$ is determined as

$$Q_n(p_\perp) = \int \exp\left(\frac{ip_\perp x}{\hbar}\right)\Psi_n^*(x)dx. \tag{14}$$

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