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Research paper

Analytical solution for clay plug swelling experiments

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ABSTRACT

Clay swelling experiments frequently involve monitoring the one-dimensional displacement with time of an initially dry clay plug as it imbibes water from a supply at its base. This article presents a new analytical solution for interpreting such experiments based on Richards' equation for flow in a partially saturated porous medium combined with a linear empirical function relating moisture ratio with void ratio. The analytical solution is described by just two parameter groups. The first parameter group describes the swelling potential of the clay. The second parameter group describes the rate at which the swelling plug reaches equilibrium, which is controlled by permeability and capillary pressure. Application of the analytical solution is demonstrated by calibration to one-dimensional displacement data from clay swelling experiments for an illite and bentonite clay.

1. Introduction

Compacted Wyoming bentonite, a clay rock composed predominantly of sodium montmorillonite, is widely used as a barrier material for nuclear radioactive waste repositories, as an adsorbent, and as an analogue for swelling clay mineral rich sub-sea shale formations (Komine and Ogata, 1994). The two-dimensional layered structure of aluminosilicate clay minerals results in a high surface area and, combined with the presence of exchangeable cations, gives rise to the adsorbent properties of bentonite for radionuclides (for example, Cs^+), and other contaminants (Eriksen et al., 1999).

These properties of bentonite and related clay minerals also result in a propensity to spontaneously hydrate upon contact with water. To accommodate this hydration, the bulk volume of the clay mineral must increase if the material is unconfined; if confined, a swelling pressure arises instead. Upon swelling, the transport properties of the hydrated clay mineral material also change due to the changes in pore size distribution.

One manifestation of swelling that presents significant challenges to oil and gas operations is shale instability. Oil and gas reservoirs are usually topped by an impervious cap rock, which holds the less dense hydrocarbons within the subterranean reservoir. In most cases, the caprock will be a clay mineral rich shale. Thus shales are invariably encountered when drilling the well-bore to access the oil reservoir. To clear the rock fragments (cuttings) from the well-bore during drilling, and to maintain hydrostatic pressure in the cleared hole and thus formation integrity, a technical drilling fluid is used. This fluid is colloquially known as “mud”. In the early days of oil exploration, oil-based muds (OBM) were generally used, based on crude oil. However, owing to the environmental impacts of OBMs, drilling engineers have focussed on developing less environmentally damaging water-based muds (WBM).

The clay minerals present in shale may either disperse (for example kaolinite) or swell (for example montmorillonite) upon contact with WBMs, causing instability in the well bore region, such as swelling/sloughing, in the medium-long term, or increased plasticity and the aggregation of shale cuttings to the drill string (“bit balling”) in the short term (Anderson et al., 2010). The swelling of clay is strongly controlled by the composition of pore-water (Chai et al., 2014; Chen et al., 2016; Chavali et al., 2017; Oren and Akar, 2017). To address these problems, the industry has tried to increase the environmental performance of WBMs through use of more sophisticated water-based technologies including small organic molecules to inhibit clay mineral/shale swelling. Compacted bentonite is frequently used as a model in which to probe the efficacy of the swelling inhibitors.

The swelling properties of clay minerals have long been studied, in aqueous brines and in organic solvents (Anderson et al., 2010), though often at the single crystal level. Clay crystal swelling has been partitioned into both crystalline and osmotic swelling regimes (Anderson et al., 2010). Studies have primarily focussed on the equilibrium swelling of the clay minerals, with the expansion of the crystal at given conditions of temperature and salinity tested against classical colloid

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theories, e.g. DLVO theory (Smalley, 2006). Though the crystal chemistry and swelling potential of clay minerals are relatively well understood (Wilson and Wilson, 2014; Anderson et al., 2010), the link between clay mineral crystal swelling and shale stability, where many components influence the bulk swelling (Van Oort, 2003), is less clear, and it has even been argued that clay mineral swelling properties are not relevant in the oil field and that shale swelling may not occur when artefacts of experiments are carefully accounted for (Santarelli and Carminati, 1995).

It is inherently challenging to create compacted shale-like materials that replicate the properties and behaviour of saturated shale under wellbore conditions. As such, compacted bentonite and other natural clays will remain in use as a proxy for shales to allow testing of drilling fluid formations and additives and it is imperative to account for the artefacts introduced during production of compacted clay mineral shale analogues to be able to discern performance improvements in new drilling fluid technologies (Santarelli and Carminati, 1995). In particular, and especially to attempt to understand bit balling, the short term swelling rates of clay minerals need to be better understood.

To address this, a non-contact one-dimensional displacement meter has been developed and used to measure the short term swelling response of compacted bentonite tablets (Sellick et al., 2017). Similar experiments in this context are also reported by Chen et al. (2016). Modeling of the swelling process can help reveal the relative importance of processes contributing to the compacted bentonite swelling. Whereas very accurate computational chemistry methods, such as atomistic molecular dynamics, have enabled swelling energetics to be probed at the clay mineral crystal scale (Suter et al., 2011), and more complex multi-scale methods have been developed (Suter et al., 2015), these are computationally very expensive. The objective of this study is to improve understanding about the kinetics of hydration processes through an analytical model, which allows rapid calculation of swelling curves to be tested against experimental data.

Our approach builds on modeling techniques previously developed to understand swelling and shrinkage of clay rich soils (Philip, 1969; Smiles and Raats, 2005; Su, 2010). Such models employ Richards' equation to describe water movement through partially saturated porous media combined with empirical relationships between moisture ratio (volume of water/total volume) and void ratio (volume of voids/total volume) (Peng and Horn, 2007). The swelling of clay gives rise to moving boundary conditions. However, these can be dealt with by using a material coordinate system, associated with a theoretical dry clay mass, as opposed to a spatial coordinate system (Philip, 1969; Su, 2010).

In the context of the swelling experiments described above, the resulting non-linear partial differential equation (PDE) have traditionally been solved numerically using finite differences (Kim et al., 1999). Although a range of analytical and quasi-analytical solutions for this class of problem have been sought in the past (Raats, 2002), these have mostly been in the context of infiltration. In this article, a linear relationship between moisture ratio and void ratio is assumed such that the resulting PDE is linear and can be solved analytically. An exact analytical solution for the one-dimensional displacement of a compacted clay tablet as a function of time is then derived. Finally, the efficacy of the analytical solution is demonstrated by calibration to experimental data sets from two different clays.

2. Mathematical model

The approach used here was originally developed by Philip (1969) in the context of swelling soils, and has been frequently applied in the context of soil-deformation modeling (Kim et al., 1999; Smiles and Raats, 2005; Su, 2010). An explanation of how this approach can be used to derive an analytical solution for clay plug swelling experiments is described as follows.

The bulk volume of an unsaturated clay mass, V_b [L^3], can be defined by

$$V_b = V_w + V_c + V_a \tag{1}$$

where V_w [L^3], V_c [L^3] and V_a [L^3] are the volumes of water (including free and adsorbed water), clay mineral (excluding all water) and air present within the bulk clay mass of concern, respectively.

The swelling of clay due to the presence of water manifests itself through a strong correlation between void ratio, e [–], and moisture ratio, ϑ (e.g. Peng and Horn, 2007; Chertkov, 2012), defined by:

$$e = \frac{V_w + V_a}{V_c} \tag{2}$$

$$\vartheta = \frac{V_w}{V_c} \tag{3}$$

A simple way to represent this correlation is to assume that e and ϑ are related by the linear function

$$e = e_I + (\vartheta_s - e_I)\left(\frac{\vartheta - \vartheta_I}{\vartheta_s - \vartheta_I}\right) \tag{4}$$

where e_I [–], ϑ_I [–] and ϑ_s [–] represent the initial void ratio, initial moisture ratio and maximum moisture ratio of a clay sample, respectively.

Consider water movement through the bulk clay associated with a mineral clay volume of δV_c. Let δQ [$L^3\,T^{-1}$] be the net rate of outflowing water. Assuming the water and mineral clay to be incompressible, the change in moisture ratio, $\delta\vartheta$, over a time-period, δt [T], is found from

$$\delta\vartheta\delta V_c = -\delta Q\delta t \tag{5}$$

such that for an infinitesimal time-period and mineral clay volume

$$\frac{\partial\vartheta}{\partial t} = -\frac{\partial Q}{\partial V_c} \tag{6}$$

2.1. One-dimensional vertical flow

Now consider a vertically oriented cylindrical clay plug encased within a rigid and impermeable sleeve. The clay is initially dried to a uniform moisture ratio, ϑ_I, and uniform void ratio, e_I. The base of the clay plug is then soaked in water at a fixed pressure whilst the top of the plug is exposed to the atmosphere. In this way, the volumetric flow rate of water, Q, can be assumed to be described by the following form of Darcy's law (Philip, 1969)

$$Q = -\frac{Akk_r}{\mu_w}\frac{\partial(P_w + \rho_w g z + P_o)}{\partial z} \tag{7}$$

where A [L^2] is the cross-sectional area of the clay plug (which is constant due to the rigid sleeve), k [L^2] is the permeability, k_r [–] is relative permeability (which is a function of ϑ), P_w [$ML^{-1}T^{-2}$] is water pressure, μ_w [$ML^{-1}T^{-1}$] is the dynamic viscosity of water, ρ_w [ML^{-3}] is the density of water, g [LT^{-2}] is gravitational acceleration, z [L] is elevation and P_o [$ML^{-1}T^{-2}$] is the so-called overburden pressure associated with the work done in swelling the clay. See Raats (2002) for further discussion with regard to the overburden pressure term.

Capillary pressure, P_c [$ML^{-1}T^{-2}$], is defined by $P_c = P_a - P_w$ where P_a [$ML^{-1}T^{-2}$] is the air pressure. Importantly, capillary pressure is a function of ϑ. Analogous to Richards' equation, assuming the air pressure to be constant throughout and capillary pressure variations to be much larger than those associated with gravity head and overburden pressure (which is very likely in clay media), Eq. (7) reduces to

$$Q = \frac{Akk_r}{\mu_w}\frac{\partial P_c}{\partial\vartheta}\frac{\partial\vartheta}{\partial z} \tag{8}$$

The relevant initial and boundary conditions are:

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