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Technical Paper

Analytical solutions for fixed-free beam dynamics in thin rib machining

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ABSTRACT

Two different analytical approaches for predicting thin rib, fixed-free beam dynamics with varying geometries are presented. The first approach uses the Rayleigh method to determine the effective mass for the fundamental bending mode of the stepped thickness beams and Castigliano's theorem to calculate the stiffness both at the beam's free end and at the change in thickness. The second method uses receptance coupling substructure analysis (RCSA) to predict the beam receptances (or frequency response functions) at the same two locations by rigidly connecting receptances that describe the individual stepped beam sections, where the receptances are derived from the Timoshenko beam model. Comparisons with finite element calculations are completed to verify the two techniques. It is observed that the RCSA predictions agree more closely with finite element results. Experiments are also performed, where the stepped beam thickness is changed by multiple machining passes, and receptance measurements are carried out between passes. The RCSA predictions are compared to experimental results for natural frequency and stiffness. Agreement in natural frequency to within a few percent is reported.

1. Introduction

It is common practice to produce monolithic metallic components with thin ribs from solid billets by machining (subtractive manufacturing). This enables complex parts with high strength-to-weight ratio to be produced without significant assembly time and cost. Application domains range from aerospace structures to laptop cases. With the recent advances in metal additive manufacturing, it is also possible to produce near net shape parts that require only minimal machining to provide the desired surface finish and dimensional accuracy. This is particularly attractive for titanium alloys due to their high material cost and low machinability. The inherent challenge with this hybrid (i.e., combined additive and subtractive) approach is machining flexible parts. The low dynamic stiffness of the thin, near net shape ribs limits both machining stability (i.e., self-excited vibration, or chatter, can occur) and part accuracy (via the surface location errors that can arise from forced vibrations) [1].

Because thin rib machining is widespread, many authors have reported modeling efforts and production strategies with the intent to improve process performance. These efforts are summarized in Table 1. While this review may not be exhaustive, it does demonstrate the significant effort that has been expended on this important technological challenge over the past two decades.

In prior work, finite element analysis has been the primary tool to model and predict the thin rib dynamics and, in many cases, the change in the rib dynamics as material is removed. In this paper, two analytical approaches are presented to describe the stiffness and natural frequency of fixed-free beams, as well as the change in stiffness and natural frequency as material is removed by milling. The specific challenge of near net shape machining, where an initially thin rib is machined to produce a thinner rib, is addressed. The advantage of an analytical approach to the system dynamics prediction is that, as the dynamics change, the machining conditions can be selected and updated at less computational expense than a full finite element solution to maximize material removal rate for the current dynamic system. Naturally, these operating parameters change as material is removed (as evidenced by the prior research efforts), so an analytical updating procedure is beneficial.

In this analysis fixed-free beams with stepped profiles are used to represent the thin ribs geometries and subsequent material removal. The paper outline follows.

- First, the two analytical models are described. Rayleigh's method is applied to determine the effective mass and Castigliano's theorem is used to find the stiffness. Together, the mass and stiffness

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Table 1
Prior research in thin rib machining.

First author	Year	Ref.	Topic
Y. Altintas	1995	[2]	The authors considered the influence of plate dynamics on the geometric accuracy of machined thin ribs.
J. Tlusty	1996	[3]	Techniques for machining thin ribs using relieved shank tooling in a series of axial passes, finishing the rib on every pass, was described.
S. Smith	1998	[4]	Tool path strategies for the machining of thin webs which rely on the support of the unmachined workpiece were investigated.
H. Ning	2003	[5]	Finite element thin rib part models were used to assess dimensional accuracy during milling.
S. Ratchev	2004	[6]	Force-induced geometric errors were predicted in thin rib machining using finite element analysis and a voxel-transformation model.
S. Ratchev	2004	[7]	An adaptive theoretical force-finite element analysis deflection model was used to predict thin rib surface errors during milling.
U. Bravo	2005	[8]	A three-dimensional stability lobe diagram was presented that considered both the part and tool frequency response functions, as well as the intermediate stages of the rib machining.
S. Ratchev	2005	[9]	Finite element models were used to predict and compensate force-induced geometric errors in machining of thin rib structures.
V. Thevenot	2006	[10]	A three-dimensional stability lobe diagram was presented that incorporated the spatial variation in the thin rib dynamics. Modal testing and finite element analysis were used to identify the thin rib frequency response functions.
I. Mañé	2008	[11]	A spindle-tool finite element model that considered the gyroscopic moment of the spindle rotor and the speed-dependent bearing stiffness was coupled to a finite element model of the thin rib part to predict milling stability.
J.K. Rai	2008	[12]	A finite element-based milling process plan verification model was presented. The effects of fixturing, operation sequence, tool path, and operating parameters were considered to predict the thin rib part deflections.
S. Seguy	2008	[13]	The authors examined the relationship between chatter instability and surface roughness for thin rib milling. Finite element models were used to describe the rib dynamics.
O.B. Adetoro	2009	[14]	Finite element and experimental frequency response functions were used to obtain stable operating parameters for thin rib machining.
W. Chen	2009	[15]	The authors considered the effect of machining deformation that occurs in the current layer on the nominal cutting depth in the next layer during thin rib milling.
L. Gang	2009	[16]	Three-dimensional finite element models of a helical tool and a thin titanium alloy (6Al–4 V) cantilever were used to predict the cutting deformation during milling.
L. Arnaud	2011	[17]	Finite element analysis was used to model the part and time domain simulation was used to predict the thin rib machining stability.
R. Izamshaw	2011	[18]	A combination of finite element and statistical analyses were used to predict part deflection during thin rib machining.
S. Smith	2012	[19]	Sacrificial structure preforms that support the part during machining, but are not a part of the finished component, were designed and tested.
A. Polishetty	2014	[20]	The trochoidal milling strategy was used for thin rib machining of titanium alloy 6Al–4 V.

give the natural frequency. Receptance coupling substructure analysis (RCSA) is also implemented to rigidly attach the two sections of the rib: a thicker base and thinner free end. This represents the beam geometry as material is removed and a section of the profile changes. The RCSA calculations predict the assembly receptance (or displacement-to-force frequency response function); the fundamental natural frequency and corresponding modal stiffness are extracted from the predicted receptance. In both cases, comparisons to finite element analysis calculations are presented.

- Second, the experimental setup and approach are described.
- Third, a comparison between experiments and RCSA predictions is provided.
- Fourth, conclusions are presented.

2. Analytical models

2.1. Raleigh method

The maximum kinetic energy for the free vibration of a continuous (distributed mass) beam can be expressed as:

$$T_{max} = \frac{1}{2} m_{eff} \dot{y}_{max}2, \tag{1}$$

where m_{eff} is the effective mass for the fundamental mode of vibration and $\dot{y}_{max}$ is the maximum beam velocity in the lateral direction (perpendicular to the beam axis) [21]. For harmonic motion, the displacement can be expressed as $y(t) = Ye^{i\omega t}$, where ω is the circular frequency (rad/s) and, therefore, the velocity is $\dot{y}(t) = i\omega Ye^{i\omega t} = i\omega y(t)$ [22]. Substituting for velocity in Eq. (1) yields:

$$T_{max} = \frac{1}{2} m_{eff} \omega^2 y_{max}2. \tag{2}$$

The beam geometry for thin rib machining is depicted in Fig. 1. The fixed-free beam profile is shown, where the thickness has been decreased at its free end by a first machining pass. With each subsequent pass, more material is removed and the profile is changed (i.e., the length of the thin section increases) until the final geometry is obtained with the desired, uniform thickness.

Eq. (2) is updated using the expression for the deflection, $y(x)$, at the beam free end due to a force at the same location:

$$T_{max} = \frac{1}{2} \frac{\left(\frac{m_1}{L_1}\int_0^{L_1} y(x)^2 dx + \frac{m_2}{L_2}\int_{L_1}^{L_1+L_2} y(x)^2 dx\right)}{y_{max}2} \omega^2 y_{max}2, \tag{3}$$

where m_1 and m_2 are the masses of the two sections from Fig. 1 and L_1 and L_2 are the lengths. The integral is split due to the step change in thickness. Equating Eqs. 2 and 3 gives the effective mass.

$$m_{eff} = \frac{m_1}{L_1}\int_0^{L_1} y(x)^2 dx + \frac{m_2}{L_2}\int_{L_1}^{L_1+L_2} y(x)^2 dx \tag{4}$$