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Micro-statistical modeling of an imperfect interface in a piezoelectric bimaterial under inplane static deformations

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ABSTRACT

A micro-statistical model is proposed for investigating the effective properties of a micro-damaged interface between a piezoelectric layer and a piezoelectric half-space under inplane electroelastostatic deformations. The interface is modeled as damaged by periodic arrays of micro-cracks. The lengths and the positions of the micro-cracks on a period interval of the interface are randomly generated. The conditions on the interfacial micro-cracks are formulated in terms of hypersingular integro-differential equations with the displacement and electrical potential jumps across the interface being unknown functions to be determined. To gain new useful physical insights into the behaviors of the imperfect interface, the influences of the material constants, the width of the layer and the crack densities of the interface on the effective properties of the interface are examined in details.

1. Introduction

For a mathematically more tractable analysis of layered materials having microscopically damaged interfaces, the interfaces may be simplified as ones with effective properties. Such an interface between two elastic materials denoted by 1 and 2 is modeled as a continuous distribution of springs with interfacial conditions given by

$$\underline{\underline{\sigma}}^{(1)} \cdot \underline{\mathbf{n}} = \underline{\underline{\sigma}}^{(2)} \cdot \underline{\mathbf{n}} = \underline{\underline{\mathbf{k}}} \cdot (\underline{\mathbf{u}}^{(1)} - \underline{\mathbf{u}}^{(2)}) \quad \text{on } \Gamma, \tag{1}$$

where Γ denotes the spring-like interface between the two elastic materials, $\underline{\mathbf{n}}$ is the unit normal vector to Γ pointing into material 1, $\underline{\mathbf{u}}^{(i)}$ and $\underline{\underline{\sigma}}^{(i)}$ are respectively the displacement and the stress in material i and the second rank tensor $\underline{\underline{\mathbf{k}}}$ characterizes the effective stiffness of Γ. For works on interfaces described by the interfacial conditions in (1), one may refer to Benveniste and Miloh [5], Fan and Sze [7], Hashin [9] and Jones and Whittier [10].

As piezoelectric composites play an increasingly important role in engineering applications (Park el al. [19,20] and Trolier-McKinstry and Muralt [25]), researchers have shown interest in the analyses of imperfect interfaces between piezoelectric materials. The interfacial conditions in (1) may be generalized to analyze weak interfaces between piezoelectric materials, as in Fan et al. [8] and Li and Lee [12]. In general, the interface between piezoelectric materials may be damaged both mechanically and electrically (Li et al. [14]), that is, not only the displacement is discontinuous across the weak interface, but the electric potential is also discontinuous. More specifically, for an interface Γ between two piezoelectric materials, the

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interfacial conditions in (1) may be generalized to

$$\left.\begin{array}{l}\underline{\underline{\sigma}}^{(1)}\cdot\underline{\mathbf{n}}=\underline{\underline{\sigma}}^{(2)}\cdot\underline{\mathbf{n}}=\underline{\underline{\mathbf{k}}}\cdot(\underline{\mathbf{u}}^{(1)}-\underline{\mathbf{u}}^{(2)})+\underline{\mathbf{b}}\,(\phi^{(1)}-\phi^{(2)})\\ \underline{\mathbf{D}}^{(1)}\cdot\underline{\mathbf{n}}=\underline{\mathbf{D}}^{(2)}\cdot\underline{\mathbf{n}}=\underline{\mathbf{c}}\cdot(\underline{\mathbf{u}}^{(1)}-\underline{\mathbf{u}}^{(2)})+w(\phi^{(1)}-\phi^{(2)})\end{array}\right\}\text{ on }\Gamma, \tag{2}$$

where $\phi^{(i)}$ and $\underline{\mathbf{D}}^{(i)}$ are respectively the electrical potential and the electrical displacement in material i and the scalar w, the vectors $\underline{\mathbf{b}}$ and $\underline{\mathbf{c}}$ and the second rank tensor $\underline{\underline{\mathbf{k}}}$ are tensorial quantities characterizing the effective properties of Γ.

Most existing research works on weak imperfect interfaces between piezoelectric materials decouple the elastic displacement and electrical potential jumps in the interfacial conditions in (2), that is, the assumption that $\underline{\mathbf{b}}=\underline{\mathbf{0}}$ and $\underline{\mathbf{c}}=\underline{\mathbf{0}}$ is made. Examples of papers making such an assumption include Chen and Lee [6], Kuo [11] , Shodja et al. [22,23], Sun et al. [24] and Zhou et al. [32]. Relatively few papers, such as Li et al. [13,14] and Shi et al. [21], consider the coupling between the mechanical and electrical imperfections in the spring-like interface. The validity of the assumption that $\underline{\mathbf{b}}=\underline{\mathbf{0}}$ and $\underline{\mathbf{c}}=\underline{\mathbf{0}}$ has not been examined in detail in the literature. A clearer idea on this can be obtained by taking into consideration the micro details of the interface. A micro-model of the interface can be developed for estimating $\underline{\underline{\mathbf{k}}}$, $\underline{\mathbf{b}}$, $\underline{\mathbf{c}}$ and w. Such a model is used in Ang et al. [2] to analyze the effective properties of micro-damaged interfaces under antiplane electroelastostatic deformations.

In the present paper, we study the effective behaviors of the interface between a piezoelectric layer and a piezoelectric half-space under inplane deformations. It (the interface) is modeled as containing periodically repeated micro-cracks, which are taken to be either electrically permeable or electrically impermeable. As in the analyses of Wang et al. [26–29] for weak interfaces between elastic materials, a statistical approach is adopted here to generate randomly the lengths and positions of the micro-cracks within a period interval of the interface. The conditions on the micro-cracks are formulated in terms of the hypersingular integro-differential equations, where both the displacement jumps and the electrical potential jumps across the interface appear as the unknown functions to be determined. Once the hypersingular integro-differential equations are solved numerically, the effective properties of the interface may be readily calculated. For specific cases, the influences of the material constants, the thickness of the layer and the crack densities of the interface on the effective properties of the interface are examined in detail. From the results, we gain new useful physical insights into the effective behaviors of the imperfect interface between the piezoelectric layer and the piezoelectric half-space under inplane electroelastostatic deformations.

2. The problem

With reference to a Cartesian coordinate system $Ox_1x_2x_3$, consider an infinitely long piezoelectric layer bonded to a piezoelectric half-space. The layer and the half-space occupy respectively the regions $0 < x_2 < h$ and $x_2 < 0$. The interface $x_2 = 0$ between the layer and the half-space is damaged by a periodic array of micro-cracks, all of which are taken to be either electrically permeable or electrically impermeable. The layer and the half-space are assumed to be perfectly bonded on the uncracked parts of the interface.

A period interval of the interface contains an arbitrary number M of arbitrarily positioned micro-cracks of possibly different lengths. More specifically, the tips of the M micro-cracks in the region $0 < x_1 < L$ are taken to be $(a^{(m)}, 0)$ and $(b^{(m)}, 0)$ $(m = 1, 2, \ldots, M)$, where $a^{(m)}$ and $b^{(m)}$ satisfy $0 < a^{(1)} < b^{(1)} < a^{(2)} < b^{(2)} < \cdots < a^{(M)} < b^{(M)} < L$. The remaining parts of the interface are periodic replicas of the region $0 < x_1 < L$. Specifically, the tips of the micro-cracks on the remaining part of the interface are given by $a^{(k)} + nL < x_1 < b^{(k)} + nL$, for $k = 1, 2, \ldots, M$ and $n = \pm 1, \pm 2, \ldots$. Refer to Fig. 1 for a geometrical sketch of the piezoelectric bimaterial for $M = 3$.

The bimaterial is subject to an inplane electroelastostatic deformation such that the Cartesian elastic displacements u_i. elastic stresses σ_{ij}, electrical potential ϕ and electrical displacements D_i are functions of only x_1 and x_2. The micro-cracks are assumed to open up and become traction-free under the electroelastostatic deformation.

If the micro-cracked interface between the piezoelectric layer and the piezoelectric half-space is simplified as a homogeneous interface with effective properties, the problem of interest here is to estimate the constant coefficients describing the effective properties of the interface.

If all the micro-cracks are electrically impermeable then the effective interface is electrically impermeable and is described by (2) which can be rewritten in Cartesian coordinates as

$$\left.\begin{array}{r}\sigma_{i2}(x_1,0^+)=\sigma_{i2}(x_1,0^-)=k_{ij}^{(\text{imp})}\Delta u_j(x_1)+b_i\Delta\phi(x_1)\\ D_2(x_1,0^+)=D_2(x_1,0^-)=c_i\Delta u_i(x_1)+w\Delta\phi(x_1)\end{array}\right\}\text{ for }-\infty<x_1<\infty, \tag{3}$$

and the constant coefficients $k_{ij}^{(\text{imp})}$, b_i, c_i and w are effective properties to be estimated. Note that $\Delta u_i(x_1) = u_i(x_1, 0^+) - u_i(x_1, 0^-)$ and $\Delta\phi(x_1) = \phi(x_1, 0^+) - \phi(x_1, 0^-)$ are respectively the displacement and electric potential jumps across the effective interface and the Einsteinian convention of summing over a repeated index is assumed for Latin subscripts running from 1 to 2.