Applied Mathematical Modelling 46 (2017) 771–784

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Theoretical and numerical aspects of nonlinear reflection–transmission phenomena in acoustics

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ARTICLE INFO

Article history:
Received 28 March 2016
Revised 1 August 2016
Accepted 10 October 2016
Available online 3 April 2017

Keywords:
Non-linear sound wave
Non-linear reflection
Non-classical absorption
Soft tissues

ABSTRACT

Equations of nonlinear acoustic wave motion in a non-classical lossy medium are used to derive generalised formulas describing the phenomena of reflection and transmission. Integral, non-local operators that are caused by the nonlinear effects in wave propagation and occur in reflection and transmission formulas are given in a form in which classical linear reflection and transmission coefficients are explicitly separated. Numerical calculations are performed for a simplified, one-dimensional wave travelling in a lossless medium. These simplifications reveal the pure effect of the impact of nonlinearities on the reflection and transmission phenomena. We consider adjacent media with different properties to illustrate various aspects of the problem. In particular, even if two media have the same linear impedance and the same material modules of the third order, we observe an explicit effect of the nonlinearity on the reflection phenomenon. The theoretical predictions are confirmed qualitatively by numerical calculations based on the finite difference time domain method.

1. Introduction

Linear models of sound propagation in non-homogeneous media (see [1]) are sufficient for explaining the reflection and transmission phenomena used in classical ultrasound imaging (USG). Images are created in the real time from the radio frequency (RF) signals coming back to a transmitter after penetrating internal body organs. Discontinuities in the acoustic properties of materials cause peaks in the RF signals that are reflected from the boundary of different tissues. Indeed the variations in the acoustic impedances of the tissues inside the human body allow the visualisation of organ boundaries in the USG images. In recent decades different methods of quantitative ultrasound imaging (QUS) or parametric imaging (PI) (see [2]) have been successfully applied to find new diagnostically valuable markers for the identification of different tissue structures. Nonlinear models are considered when high intensity ultrasound, particularly high intensity focused ultrasound (HIFU), is studied (for example, in ultrasound surgery, hyperthermia, lithotripsy, the excitation of microbubble contrast agents, and elastography imaging (shear wave elastography) [3–5]). The nonlinearity of the behaviour of media causes the high pressure portion of the wave to travel faster than the low pressure portion, resulting in the distortion of the shape of the wave. This change in waveform leads to the generation of harmonics, multiples of the fundamental or transmitted frequency, from the tissue. A tissue harmonic image often has better resolution than a conventional ultrasound image, see [6]. The standard time domain partial differential equation models can describe the effect of the nonlinearities

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http://dx.doi.org/10.1016/j.apm.2017.03.057
0307-904X/

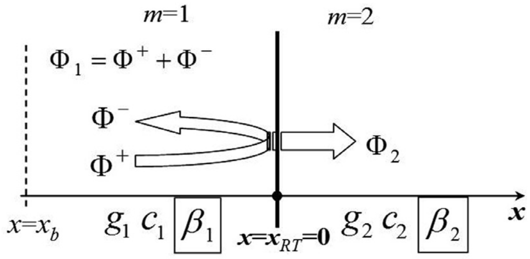

Fig. 1. The waves in the two-layered media.

of wave propagation and the effect of classical viscous wave attenuation. The classical absorption model is based on the inclusion of viscosity and thermal conduction in the equations of propagation, to yield an acoustic absorption term that is proportional to the frequency squared. However, the absorption mechanisms in soft biological tissue are significantly more complex relations. They lead to experimentally observed attenuation in the form of a power law of the frequency with an exponent ranging from 0.5 to 2 (see [7]). Nonlinear wave propagation in a medium with anomalous attenuation has been modelled by Kuznecovs type of nonlinear equation with the special non-classical absorption term in [8]. This non-classical absorption term has been proposed in the form of an operator that is non-local in space and in time. Alternatively, fractional acoustic wave equations can be applied to model the non-classical absorption phenomena [9–11]. Here, the reflection and transmission of nonlinear acoustic waves propagating in a lossy medium is based on the results given in [12] and [13], and is studied in the recently published papers [14] and [15]. The generalised reflection/transmission coefficients having the form of reflection and transmission operators are retained. To begin with, we discuss all the physical requirements needed to obtain the reflection/transmission operators, which repeats previous work. In addition, we use the uniform description of the fluid and solid continuum. Next, for one-dimensional plane wave propagation, we derive the reflection and transmission operators. We also introduce the explicit dependence on the Mach number and on the norm of the non-local absorption operator in all the relationships under consideration. Finally, we perform numerical experiments to explain the qualitative and quantitative character of the nonlinear reflection and transmission phenomena. Some of the results of this work were presented by Janusz Wojcik and Barbara Gambin at the 13th International Conference on Dynamical Systems Theory and Applications, December 7–10, 2015, in Łódź, Poland and were published in [15].

2. Modelling of reflection and transmission

2.1. Basic relations

Wave propagation in linear solids/fluids remaining in a thermodynamic equilibrium state is uniquely described with two a priori known physical properties, namely the density and the elasticity of the medium. The elasticity properties are defined by a tensor (a fourth-order elasticity tensor with specified symmetries and a second-order tensor with spherical symmetry, for solid and fluid media, respectively), whereas the density is a scalar for a solid as well as for a fluid medium. If these properties are constant in time and space then the wave propagates in a homogeneous elastic continuum, which, in the case of a solid, can generally be anisotropic. Discontinuities in density and/or elasticity defined on the boundary between two different adjacent continua lead to the reflection/transmission phenomena. If the displacements from the equilibrium position cannot be considered infinitely small (which is the linear case) and/or the reaction to the applied forces is no longer proportional to the gradient of displacements, we move to nonlinear theories of continua. We restrict our study to a medium disturbed by a plane longitudinal acoustic wave, under the assumption that the displacements u of the material particles compared to the wavelength l_b are estimated by the following inequality $u/l_b \le v_b/(2\pi\nu_b l_b)$, where ν_b, v_b are the characteristic frequency and velocity of the wave, respectively, which should be a priori known and are defined in the fixed space position, denoted symbolically by the point x_b in the one-dimensional case depicted in Fig. 1. In addition, we have $v_b/(2\pi\nu_b l_b) = P_b/(\rho_0 c_0 2\pi\nu_b l_b) = q/(2\pi) \equiv P_b/(2\pi\rho_0 c_0^2) = v_b/(c_0 2\pi)$, where l_b denotes the wavelength, $c_0 = \nu_b l_b$ the speed of sound and ρ_0 the density of the medium, respectively and q the acoustic Mach number. The elasticity of the medium in the considered case, given by Lame constants λ_0 and $2\mu_0$, is linked to the wave speed by $\rho_0 c_0^2 = \lambda_0 + 2\mu_0$. The assumption about the weak reaction of the medium to motion, or the pressure level estimation, is stated as follows:

$$P_b; v_b \equiv \max_t \left(\left| P_b(t') \right|; \left| v_b(t') \right| \right),$$

where $P_b(t)$ is the characteristic pressure at a fixed characteristic point (for example at the source location) of the disturbance. The above estimation is also valid for any more spectrally complex acoustic disturbances. The generation of acoustic disturbances with a Mach number of order $q \sim 0.01$ or greater requires sources or focusing systems that produce a high intensity of power or a high pressure level, for gases of 0.001 MPa, for liquids of tens of MPa and for solids of a few GPa. In what follows continuous media are regarded as nonlinear in reaction to acoustic disturbances (which are gradients of the scalar potential only). Below we introduce the explicit dependence in all relations on the Mach number q. The initial