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Modeling and vibratory characteristics of a mass-carrying cable system with multiple pulley supports in span range

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ABSTRACT

A theoretical dynamic model of a multi-segment mass-carrying cable system with multiple pulley supports is derived. Unlike many existing analytical cable models, the relative movement between the cable and pulley supports is considered here. The cable is modeled as a combination of a number of segments connecting the mass and the pulley supports. The Hamilton principle is used to account for the relative moving boundaries between the cable and the pulley supports. From the continuity conditions, boundary conditions and force equilibrium conditions, the non-dimensional unforced undamped characteristic equation is obtained. The natural vibratory characteristics for this kind of cable are also determined. The effects of the mass ratio and sag-to-span ratio on the frequency spectra are studied. The Pareto chart of the fundamental frequency of the multi-segment cable is examined. The results show that the frequencies are either sensitive to or independent of non-dimensional parameters in certain parameter ranges. Veering phenomena also appear between the frequencies of two approaching modes.

1. Introduction

Due to their load carrying ability while crossing long spans, cables are widely used in engineering (e.g., as power transmission lines, ocean mooring systems, satellite antenna, guyed masts, and bearing members of the cable-stayed bridges). It is then of no surprise that many researchers investigated the static and dynamic behavior of such systems [1–5].

In the reported references, single-DOF [6–9] or multiple-DOF [10,11] models were considered, depending on the resonant forms (e.g., 2:1 [12–14], 1:1 [15], 3:1 [16] internal resonances or multiple internal resonance [17–21]) and the undergoing planar/spatial motions. Small amplitude deck/tower motions of cable-stayed bridges or guyed masts can cause large amplitude cable motions, which make the study of the boundary excitations another concern [22–26]. In general, these papers focus on the nonlinear behavior of the structure with fixed ends or excited boundaries. In these works, the cables were considered single-segment and the relative movements between the support and the cable were not taken into account.

In contrast, cables with multiple pulley supports in the span range are designed to transport much heavier masses. For instance, cable-crane systems [27,28] are used as a significant temporary structure in the construction of the long-span arch bridge. This cable system involves relative movements between the pulley supports and the cable and can be modeled

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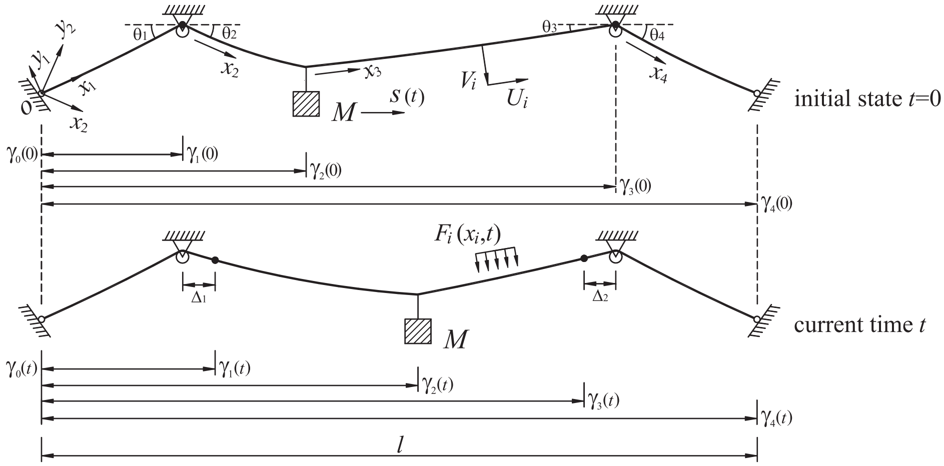

Fig. 1. Model of a mass carrying cable with two pulley supports at span range.

as multi-segment cables dynamically connected by the pulleys and the mass. In this case, the single-segment cable models [6–26] or the two-segment cable models [29] that focus on the fixed ends or excited boundaries are not adequate to consider the relative motions between the supports and the structure for this system. Moreover, the methods in references [30–35], where the connections between mass and cable are generally treated as a friction [32,34] or a moving oscillator [35], are unsuitable for this kind of cable. Numerical methods to simulate cable systems with multiple segments also exist in the literature [36–38], but do not make the scope of this current paper. In [36], a framework that combines Lagrangian and Eulerian nodes is proposed to simulate compound pulley systems. However, only nodal velocity based viscous damping can be included in its current implementation. In [37], a two-layer model is proposed to simulate hoisting cables in real time. However, the cable gravity is not considered in its first-layer model. In [38], a hybrid model that combines the quasistatic massless cable and the multibody chain approach is employed to simulate wires contacting with rigid multibodies.

In this paper, we introduce an analytical model for multiple-segment mass-carrying cable with multiple pulley supports in the span range. Unlike existing analytical cable models, the relative movement between the cable and pulley supports is considered here. *Simultaneous consideration of the attached mass and the relative motion between the pulley supports and the cable is very challenging in analytical approaches. Therefore, accurate analytical modeling of multiple-segment long-span suspended cables accounting for such effects is important and constitutes the motivation for this work.* The cable is dynamically divided into multiple segments by the pulleys and the moving mass in the global system. The Hamilton principle is used to account for the relative motions between the pulleys and the cable. Two contributing factors to the total displacement are considered: the dynamic deformation induced by the moving of the attached mass and the global vibration of the cable segments.

In Sections 2 and 3, we introduce the nonlinear dynamic equation of the mass-carrying multiple-segment cable and its natural vibratory characteristics. A parametric study of the influence of the mass ratio and the sag-to-span ratio on the vibratory characteristics of the cable is presented in Section 4. The linear natural frequencies are either sensitive to or independent of the structural parameters in specific intervals. Multiple veering phenomena are frequently observed between different modes, which indicates that this cable system may have high risk of encountering internal resonances that can induce unexpected severe vibrations. These results can provide useful guidance on the preliminary design of the cable systems with multiple pulley supports. For example, cable-crane systems are commonly used to facilitate the construction of long-span arch bridges. It is crucial to design a safe and reliable cable-crane system that does not display unexpected internal resonances. The identification of the vibratory characteristics of a cable-crane system can guide the selection of system parameters (e.g., the weight, sag, or length of cable segments) outside the unsafe region to avoid unexpected accidents in the construction of long-span arch bridges. The approach used here considering the relative motion between the cable and supports in the span range can be extended to other similar problems.

2. Equations of motion

In this section, the planar dynamic model of the multi-segment cable with an attached mass is obtained. A mass-carrying cable system with two pulley supports in the span range is shown in Fig. 1. It is separated into four segments by the frictionless pulley supports and attached mass. l is the total span of the cable, and $O-x_iy_i$ is the local coordinate system of each cable segment. Here O is the common origin for all local coordinates, x_i is parallel to the connection line of the two ends of the ith cable segment, y_i is perpendicular to x_i. θ_i is the inclination angle of the straight connection line of the two ends of the ith cable segment measured with respect to the horizontal line in the global coordinate system. To avoid overcrowding, only x_1-y_1 and x_2-y_2 are illustrated at the origin O in Fig. 1. For the other segments, we just indicate the