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Method of particular solutions using polynomial basis functions for the simulation of plate bending vibration problems

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ABSTRACT

The traditional polynomial expansion method is deemed to be not suitable for solving two- and three-dimensional problems. The system matrix is usually singular and highly ill-conditioned due to large powers of polynomial basis functions. And the inverse of the coefficient matrix is not guaranteed for the evaluation of derivatives of polynomial basis functions with respect to the differential operator of governing equations. To avoid these troublesome issues, this paper presents an improved polynomial expansion method for the simulation of plate bending vibration problems. At first, the particular solutions using polynomial basis functions are derived analytically. Then these polynomial particular solutions are employed as basis functions instead of the original polynomial basis functions in the method of particular solutions for the approximated solutions. To alleviate the conditioning of the resultant matrix, we employ the multiple-scale method. Numerical experiments compared with analytical solutions, solutions by the Kansa's method, and reference solutions in references confirm the efficiency and accuracy of the proposed method in the solution of Winkler and thin plate bending problems including irregular shapes.

1. Introduction

In recent decades, simulations of plate bending vibrations are of considerable technological importance to the product design process [1–3]. The plate bending problems have consequently attracted considerable attentions from scientific and engineering aspects, and varieties of approximate methods have been developed. The use of mesh such as the finite element method (FEM) [4,5] is a basic characteristic of the traditional approaches used for dynamic simulation of bending problems. In these methods, approximations of dynamic field variables are made within each element, which need a discretization of the whole structure into small elements. Compared with the FEM, the boundary-type methods such as the boundary element method (BEM) [6,7] which forms an integral equation, a boundary mesh is also required to obtain a numerical prediction. It is recognised that the meshing for 3D structures with complex geometries is an arduous, time consuming, and expensive task. Hence, considerable interests have been received to avoid or simplify the meshing task.

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In contrast to the mesh-based methods, the meshless methods have been a goal in the computational mechanics community and receive great attentions in the past several decades, for example, the smoothed particle hydrodynamics [8,9], the diffuse element method [10,11], the element-free Galerkin method [12], and the radial basis function collocation method [13–16], etc. There are other types of meshless methods, named the meshless boundary-type methods, such as the Trefftz method [17,18], the method of fundamental solutions [19–22], the regularized meshless method [23,24], the wave based method [25,26], the boundary node method [27–31], and the boundary point interpolation method [32,33], ect. These techniques can be classified into two distinct concepts. The first family of methodologies seek the approximation with the expansion functions which satisfy the homogeneous governing equations a priori. Then, the unknown weight parameters are obtained by requiring the approximations satisfying the given boundary conditions. The wave based method and the method of fundamental solutions, etc, belong to this family. The disadvantage of this family of meshless methods is that trial functions which are solutions of the governing equations must exist so that an approximate solution may be obtained. The second methodology attempts to relax on those strong requirements, and tries to find the approximation by using some non-trivial basis functions such as the method of particular solutions (MPS) [34]. The MPS has been proved to be flexible, accurate and easy to use. In the MPS, derivation of the particular solutions for a given differential equation is crucial to success of this method. And it is known to all that the particular solution is not unique and there are numerous ways to obtain particular solutions for various problems. Over the past two decades, much effort has been devoted to finding the particular solution of the problems using different basis functions. Among them, the radial basis functions (RBFs) have witnessed a research boom and been successfully employed to obtain the particular solution for some certain partial differential equations [35,36]. However, there are still some challenges concerning the determination of the optimal parameter in the RBFs. To avoid such difficulties, the particular solutions based on the Chebyshev polynomial functions have been evaluated and adopted as alternative approaches [37,38]. However, the solution procedure is tedious. The forcing term of the differential equation should be smoothly extendable to the exterior of the solution domain in the case of irregular domains. And it remains a difficult task to evaluate closed-form particular solutions for general differential operators.

Recently, we propose a new strategy for obtaining particular solutions using standard polynomial basis functions. Then the proposed particular solution is coupled with the MPS for the simulation of partial differential equations [39]. The main advantage of the polynomial basis function over the Chebyshev polynomial functions is that the collocations can be made arbitrarily inside the solution domain. In this paper, we extend this method to the analysis of plate bending vibration problems. To avoid the ill-conditioning of the resultant matrix, the multiple-scale method [40,41] which is a pre-conditioning technique is used to reduce the condition number of the resulting system. It is noted here that one of the weakness of the primary algorithm proposed in [39] is the restriction of $\lambda \neq 0$ in Eq. (1). In this paper, the proposed method is further extended to more general partial differential equations by the subtracting and adding-back technique relaxing the restriction on this condition.

The rest of this paper is organized as follows. In Section 2, the governing equation and corresponding boundary conditions of plate bending vibration problems are presented. In Section 3, we present a detailed implementation of the MPS in the context of the polynomial particular solutions for plate bending vibration problems. In Section 4, numerical results of the proposed method are compared against analytical solutions, reference solution by the FEM, the Kansa's method, and reference solutions in literatures for a series of different supported edges with regular and irregular domain geometries. Finally, some conclusions are outlined in Section 5.

2. Problem definition

Without loss of any generality, the governing equation of the plate bending problems based on the thin plate theory for not too high frequencies can be simplified as follows:

$$\nabla^4 w_z(x,y) + \lambda w_z(x,y) = \frac{q(x,y)}{D}, \ (x,y) \in \Omega, \tag{1}$$

where $w_z(x, y)$ represents the steady-state out-of-plane displacement, $q(x, y)$ is loading of the force, and λ denotes the given function which is defined by the type of plates, as follows:

$$\lambda = \frac{k_w}{D}, \tag{2}$$

for the Winkler plate, and

$$\lambda = -k_b^4, \tag{3}$$

for the thin plate, where k_w denotes the foundation stiffness. The plate bending wave-number k_b and the plate bending stiffness D are defined as follows:

$$k_b = \sqrt[4]{\frac{\rho h \omega^2}{D}} \text{ and } D = \frac{Eh^3}{12(1-\mu^2)}, \tag{4}$$

with E the Young's modulus, μ the Poisson's ratio, ρ the plate material density, ω the angular frequency, and h the plate thickness. In order to simulate Eq. (1), two boundary conditions at each node on the boundary should be specified. Below formulations are the commonly encountered boundary conditions at the plate boundary: