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Viscoelastic body colliding against a rigid wall with and without dry friction effects

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ABSTRACT

The impact of a body against a rigid wall is studied. The body consists of a rigid block and a viscoelastic rod described by the distributed-order fractional model and in particular its solid-like and fluid-like special cases. Translatory motion of a body is studied in two cases: without and with the influence of dry friction. When present, dry friction is modeled by the Coulomb friction law. The problem is treated analytically by the use of the Laplace transform method and solutions are obtained in a convolution form.

1. Introduction

The impact of an initially undeformed body against a rigid wall is studied. The term body is used to refer to a system consisting of a light viscoelastic rod of length L attached to a rigid block of mass m. The body slides translatory (in the first case without and in the second case with the presence of dry friction) on the horizontal surface along the x-axis, which coincides with rod's axis. At the initial moment, having velocity v_0, the body, or more precisely viscoelastic rod, comes into contact with the wall. The rectangular coordinate system is chosen so that its origin coincides with the point where rod is attached to the block at the initial time instant, see Fig. 1. Described type of impact corresponds to a Hertz-type theory, see [1]. This problem, without the presence of dry friction, is treated analytically in [2] and numerically in [3], where the mechanical properties of a viscoelastic rod are described by the fractional Zener constitutive equation. Also, if the friction force, modeled by the Coulomb friction law, see [4,5], is taken into account, this problem is treated numerically in [6], by the use of the Grü nwald–Letnikov scheme together with the slack variable method. The slack variable method is used in order to determine the time-instant when the body changes the motion phase. Similar problem of impact of two bodies (with the dry friction taken into account) is studied in [7]. An overview of impact responses in viscoelastic systems is given in [8].

The aim of this paper is to generalize the problem of collision of a body against a rigid wall by changing the constitutive equation of the viscoelastic body. Also, the case of frictionless sliding of a body, as well as the case when the body slides in the presence of dry friction will be considered. Following [9], the system of equations, corresponding to the problem, will be solved analytically using the Laplace transform method and solutions will be obtained in the convolution form.

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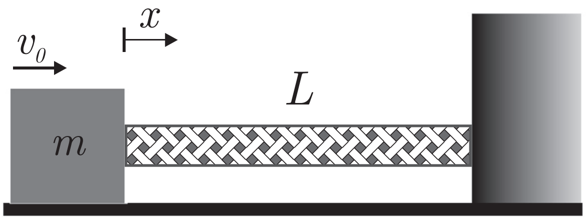

Fig. 1. Body consisting of a rigid block and a viscoelastic rod.

During the impact there are two different types of energy dissipation mechanisms. The first one is due to the deformation of a rod and its material properties. The second one is due to the presence of a dry friction between the block and the surface.

The material properties of the rod are modeled by the general distributed-order constitutive equation

$$\int_0^1 \phi_\sigma(\gamma)\, {}_0^{\mathrm{C}}\mathrm{D}_t^\gamma \sigma(t)\mathrm{d}\gamma = \int_0^1 \phi_\varepsilon(\gamma)\, {}_0^{\mathrm{C}}\mathrm{D}_t^\gamma \varepsilon(t)\mathrm{d}\gamma, \quad t>0, \tag{1}$$

where σ stands for stress, ε for strain and ϕ_σ, ϕ_ε are constitutive functions or distributions. The operator of Caputo fractional derivative of order $\gamma\in(0,1)$, ${}_0^{\mathrm{C}}\mathrm{D}_t^\gamma$, is defined by

$${}_0^{\mathrm{C}}\mathrm{D}_t^\gamma y(t) = \frac{t^{-\gamma}}{\Gamma(1-\gamma)} * \dot{y}(t) = \frac{1}{\Gamma(1-\gamma)}\int_0^t \frac{\dot{y}(\tau)}{(t-\tau)^\gamma}\mathrm{d}\tau, \quad t>0, \tag{2}$$

with $\dot{y}(t)=\frac{\mathrm{d}}{\mathrm{d}t}y(t)$ and $*$ denoting the convolution: $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)\mathrm{d}\tau$. For the detailed account on the fractional calculus see [10]. Constitutive equation of distributed-order, similar to (1), used for modeling both viscoelastic and dielectric media, is analyzed in [11]. Assuming constitutive functions/distributions in different forms, one recovers constitutive equations of linear viscoelasticity. For example, if the constitutive distributions are assumed as

$$\phi_\sigma(\gamma)=\delta(\gamma)+\tau_\sigma\,\delta(\gamma-\alpha), \quad \phi_\varepsilon(\gamma)=E(\delta(\gamma)+\tau_\varepsilon\,\delta(\gamma-\alpha)),$$

the fractional Zener model

$$\sigma(t)+\tau_\sigma\, {}_0^{\mathrm{C}}\mathrm{D}_t^\alpha\sigma(t) = E\big(\varepsilon(t)+\tau_\varepsilon\, {}_0^{\mathrm{C}}\mathrm{D}_t^\alpha\varepsilon(t)\big), \quad t>0, \tag{3}$$

where $\alpha\in(0,1)$, $E>0$, $\tau_\varepsilon>\tau_\sigma>0$, is obtained. For historical view point concerning the fractional Zener model see [12–14]. Restrictions on model parameters: generalized Young modulus E, generalized retardation time τ_ε and generalized relaxation time τ_σ, follow from the Second law of thermodynamics, see [15,16]. The fractional Zener model (3) is often used in modeling the behavior of viscoelastic materials, see for example [17,18]. The behavior of viscoelastic materials of fractional order in wave propagation phenomena is studied in [19].

In numerical examples, two constitutive models will be treated:

$$\int_0^1 a^\gamma\, {}_0^{\mathrm{C}}\mathrm{D}_t^\gamma\sigma(t)\mathrm{d}\gamma = \int_0^1 b^\gamma\, {}_0^{\mathrm{C}}\mathrm{D}_t^\gamma\varepsilon(t)\mathrm{d}\gamma, \quad t>0, \tag{4}$$

$$\sigma(t)+a\, {}_0^{\mathrm{C}}\mathrm{D}_t^\alpha\sigma(t) = b_0\, {}_0^{\mathrm{C}}\mathrm{D}_t^{\beta_0}\varepsilon(t)+b_1\, {}_0^{\mathrm{C}}\mathrm{D}_t^{\beta_1}\varepsilon(t), \quad t>0, \tag{5}$$

with thermodynamical restrictions $0<a<b$ and $a,\ b_0,\ b_1>0$, $0<\alpha<\beta_0<\beta_1<1$, respectively. The first one is the distributed-order model of the solid-like viscoelastic body and the second one is the six-parameter model of the fluid-like viscoelastic body. These models are obtained from the general distributed-order model (1) if constitutive functions/distributions ϕ_σ and ϕ_ε are chosen as

$$\phi_\sigma(\gamma)=a^\gamma, \quad \phi_\varepsilon(\gamma)=b^\gamma, \tag{6}$$

$$\phi_\sigma(\gamma)=\delta(\gamma)+a\,\delta(\gamma-\alpha), \quad \phi_\varepsilon(\gamma)=b_0\,\delta(\gamma-\beta_0)+b_1\,\delta(\gamma-\beta_1), \tag{7}$$

respectively. Note that the difference between solid and fluid-like materials is observed in the creep test. The test consists in subjecting rod's free end to a force given by the Heaviside function, i.e., sudden, but later constant force. Then, for infinite time, the rod made of solid-like material creeps to a finite value of displacement, while the rod made of fluid-like material creeps to an infinite value of displacement.

In case when the dry friction is taken into account, it is modeled by the Coulomb friction law so that in the slip phase, i.e., when block moves translatory with (relative) velocity $v_x=\dot{x}$ along the dry surface, the friction force acting on the block is constant and opposite to the direction of the velocity. In the stick phase, i.e., while the block does not move, the friction force balances external forces acting on the body up to the limiting value of friction force, which is equal to its constant value in the slip phase. The stick phase, according to [5], is well described by this simple model and that is the reason for