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Vibration analysis of conveying fluid pipe via He's variational iteration method

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ABSTRACT

In this paper, a recently new semi-analytical method, i.e., He's variational iteration method is developed to apply to free vibration analysis of conveying fluid pipe. The critical flow velocity and frequency of pipe conveying fluid are obtained with considering the various boundary conditions. The results are compared with the ones of different transform method, and prove VIM that has the same precision and efficient with DTM. The mode shapes of cantilevered pipe and both ends with elastic support pipe are shown under different flow velocity.

1. Introduce

Vibration analysis of conveying fluid pipe has been studied extensively in the past few decades. Conveying fluid pipe is as important components of some engineering structures, such as oil pipelines, nuclear reactor, marine risers, heat exchanger and so on [1]. The linear dynamics of pipe conveying fluid has been understood for some time, which can be seen in the monograph [2]. As far, vibration problems for pipe conveying fluid have been analyzed by the different approaches. Paidoudssis and Issid [3] showed the liner dynamic of pipe conveying fluid by the Galerkin method. Galerkin method has been applied extensively in the linear and nonlinear analysis of pipe conveying fluid [4]. Qian et al. [5] studied the instability of simply supported pipes conveying fluid under thermal loads is studied by the DQM. Ni et al. [6] analyzed the free vibration problem of pipes conveying fluid with several typical boundary conditions by the DTM. Pramila [7] analyzed vibration of pipe conveying fluid by the FEM. Kuiper [8] applied the D-decomposition method to vibration analysis of pipe conveying fluid. Gu [9] showed the dynamic response of a clamped–clamped pipe conveying fluid by the generalized integral transform technique (GITT). Xu et al. [10] deserved natural frequencies of fluid conveying pipes using homotopy perturbation method. Precise integration method (PIM) was used by Liu and Xuan [11] to analyze supported pipes conveying pulsating fluid. Li and Yang [12] showed the dynamical response of the pipe conveying fluid under forced vibration by the Green function method. From the published literatures about pipe conveying fluid, application of new methods to research of pipe conveying fluid has always been important research area. Based on the front statement, there are no previous study endeavored to perform vibration analysis of pipe conveying fluid based on the He's vibrational iteration method (VIM). This method was developed by the Chinese mathematician He [13] as a modification of a general Lagrange multiplier method. Vibrational iteration

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Table 1
Eigenvalue $\mathrm{Im}\omega_i$ for k up to 10 approximations for pinned–pinned pipe for $u = 0.0$.

k	$\mathrm{Im}\omega_1$	$\mathrm{Im}\omega_2$	$\mathrm{Im}\omega_3$	$\mathrm{Im}\omega_4$
1	9.8902	28.2141		
2	9.8697	37.2824		
3	9.8696	39.4169		
4	9.8996	39.4779		
5	9.8696	39.4784	88.9694	128.1751
6	9.8696	39.4784	88.8280	151.0710
7	9.8696	39.4784	88.8264	157.6386
8	9.8696	39.4784	88.8264	157.9100
9	9.8696	39.4784	88.8264	157.9137
10	9.8696	39.4784	88.8264	157.9137

method is relatively new technique, which is a powerful tool for solving various kinds of problems [14]. Tatari [15] proposed the improvement that accelerates the convergence of the sequences which result from the variational iteration method for solving systems of differential equations. Liu and Gurram [16] presented a way of using He's variational iteration method to solve free vibration problems for an Euler–Bernouli beam under various supporting conditions. Chen et al. [17] applied vibrational iteration method to analyze free vibration problem of the rotating tapered Timoshenko beam. Younesian et al. [18] deserved analytical solutions for free oscillations of beams on nonlinear elastic foundations using the variational iteration method. Huang et al. [19] provided a new modification of the variational iteration method (MVIM) for solving van der Pol equations. Malvandi [20] applied variational iteration method coupled with Pad-approximation (VIM-Pad) that attained a reliable mathematical expression for amperometric enzyme kinetics problems. He et al. [21] pointed out that the so called enhanced variational iteration method [22] for a nonlinear equation arising in electrospinning and vibration-electrospinning process is the standard variational iteration method. Malvandi [23] studied the unsteady motion of a rigid spherical particle in a quiescent shear-thinning power-law fluid by coupling the homotopy-perturbation method (HPM) and the variational iteration method (VIM). Chen [24] re-examined free vibration of marine risers by means of variational iteration method.

The aim of this paper presents the application of VIM to research about conveying fluid pipe. The rest of this paper is organized as follows. In Section 2, the theoretical background and improvement of the VIM is briefly stated. In Section 3, the equation of motion of pipes conveying fluid with various boundary conditions is given. In Section 4, the variational iteration formulation of solution of pipe conveying fluid is derived. Solution procedures of variational iteration method are also presented in Section 4. In Section 5, numerical results are presented and solutions are compared with results published in the literature. Mode shapes of pipe conveying fluid under different flow velocity are shown in two boundary conditions. Conclusions are given in the last section.

2. He's variational iteration method (VIM)

In this section, the concept of variational iteration method is introduced. We consider the following general differential equation:

$$Lu(t) + Nu(t) = g(t), \tag{1}$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a forcing term. According to He's variational iteration method [13,14,25,26], we can construct the following correction function as:

$$u_{n+1} = u_n + \int_0^x \lambda(s)(Lu_n(s) + N\tilde{u}_n(s) - g(s))ds, \tag{2}$$

where λ is a general Lagrange multiplier, which can be determined optimally via using the variational theory [27]. The subscripts n denote the nth approximation; $\tilde{u}_n$ is considered as a restricted variation i.e., $\delta\tilde{u}_n = 0$. The detailed descriptions of variational iteration method and its applicability in solving various kinds of differential equations are given by He [25] and He and Wu [26]. Iteration formulae and related Lagrange multipliers for solving several typical differential equations are listed in Table 1 of the documents [18].

Recently, Tatari and Dehghan [15] improved He's variational iteration method for solving the higher order differential equations and complexity of the VIM was decreased considerably. The higher order differential equations of order n is as follows:

$$\frac{d^n y}{dt^n} + f(y, y', \ldots, y^{n-1}) = g(t) \tag{3}$$