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Short communication

Note on continuous interval-valued intuitionistic fuzzy aggregation operator

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ABSTRACT

Recently, Zhou et al. proposed the C-IVIFOWA operator for aggregating interval-valued intuitionistic fuzzy numbers. The flaws of the C-IVIFOWA operator are illustrated by two counter-examples. The aim of this short communication is to show how the existing aggregation operator can be improved to increase accuracy. Some desirable properties of the revised C-IVIFOWA operator are studied in detail.

1. Introduction

To choose the best alternative, decision makers need to aggregate their preference information by some proper aggregation operators. A common used aggregation operator is the ordered weighted averaging (OWA) operator, which is introduced by Yager [1]. The OWA operator contains a parameterized family aggregation operators, such as the maximum, minimum and simple arithmetic mean. Since its proposed, the OWA operator has been widely used in many practical areas [2–5]. Due to the increasing complexity of modern society, the preference information provided by decision makers often express as interval numbers. To aggregate all the values in a closed interval, Yager [6] introduced the continuous ordered weighted averaging (C-OWA) operator, which is an effective extension of the OWA operator. Enlightened by the idea of C-OWA operator, the continuous ordered weighted geometric averaging (C-OWGA) operator [7], the continuous ordered weighted harmonic averaging (C-OWHA) operator [8], the continuous generalized ordered weighted averaging (C-GOWA) operator [9] and the continuous quasi-ordered weighted averaging (C-QOWA) operator [10] are proposed respectively.

In [11], Atanassov introduced the intuitionistic fuzzy set, which is described by a membership function and a non-membership function. The intuitionistic fuzzy set reflects the positive and negative information given by decision makers, and is very suitable for representing fuzzy information in decision making problems [12]. With the increasing vagueness of the real world, the decision makers' preference is becoming more and more uncertain. Consequently, the decision makers' preference is provided in the form of interval-valued intuitionistic fuzzy numbers instead of the intuitionistic fuzzy numbers [13,14]. Namely, the membership degree and non-membership degree of interval-valued intuitionistic fuzzy number both are expressed as interval numbers. Many aggregation operators have been developed to aggregate interval-valued intuitionistic fuzzy information, such as interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator [15], interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator [15], interval-valued intuitionistic fuzzy ordered weighted

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averaging (IIFOWA) operator [16], interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator [16], interval-valued intuitionistic fuzzy hybrid averaging (IIFHA) operator [17], interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator [17], induced generalized interval-valued intuitionistic fuzzy hybrid Shapley averaging (IGIIFHSA) operator [18], interval-valued intuitionistic fuzzy prioritized (IVIFP) operator [19] and interval-valued intuitionistic fuzzy Einstein hybrid weighted geometric (IVIFEHWG) operator [20]. The above-mentioned aggregation operators all focus on the endpoints of intervals, values in the membership interval and non-membership interval are not completely considered. To overcome this problem, Zhou et al. [21] proposed a continuous interval-valued intuitionistic fuzzy ordered weighted averaging (C-IVIFOWA) operator based on the C-OWA operator. The C-IVIFOWA operator aggregates all the values in the closed membership interval and non-membership interval. However, the C-IVIFOWA operator fails in boundary accessibility and monotonicity with respect to attitudinal character. As is well known, a reasonable aggregation operator should satisfy above essential properties. Therefore, it is necessary to define an improved aggregation operator to overcome these shortages. In this paper, we point out the flaws of C-IVIFOWA operator through two counter-examples, and show how the existing aggregation operator can be improved to increase accuracy. Some desirable properties of the revised C-IVIFOWA operator are studied, such as boundedness, monotonicity based on order relations, identity and monotonicity with respect to BUM function. These properties show that the shortages of the C-IVIFOWA operator can be overcome by the revised C-IVIFOWA operator effectively.

The rest of this paper is organized as follows. Section 2 reviews some basic concepts of intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set and the C-OWA operator. In Section 3, the flaws of the C-IVIFOWA operator are illustrated by two counter-examples. The improved definition of the C-IVIFOWA operator and its desirable properties are proposed. The application of the revised C-IVIFOWA operator in project selection problem is provided in Section 4. Section 5 summarizes the main conclusions of this paper.

2. Preliminaries

In the following, some basic concepts on intuitionistic fuzzy set [11], interval-valued intuitionistic fuzzy set [13] and the C-OWA operator [6] are reviewed to facilitate future discussions.

Definition 1. Let a set X be fixed, an intuitionistic fuzzy set (IFS) A in X is defined as follows:

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \tag{1}$$

where the functions μ_A: $X \to [0, 1]$ and ν_A: $X \to [0, 1]$ determine the membership degree and non-membership degree of $x \in X$, respectively. For each $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$. The pair $\alpha = (\mu_\alpha(x), \nu_\alpha(x))$ is called an intuitionistic fuzzy number (IFN) and can be simply denoted as $\alpha = (\mu_\alpha, \nu_\alpha)$, where $\mu_\alpha, \nu_\alpha \in [0, 1]$ and $\mu_\alpha + \nu_\alpha \le 1$. For any two IFNs $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$, the following operational laws are introduced by Xu [22]:

$$\alpha \oplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha \mu_\beta, \nu_\alpha \nu_\beta), \tag{2}$$

$$\theta \cdot \alpha = (1 - (1 - \mu_\alpha)^\theta, \nu_\alpha^\theta), \quad \theta > 0. \tag{3}$$

Moreover, Xu and Yager [23] proposed the following simple method to compare IFNs.

Definition 2. Let $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ be two IFNs, then

(I) If $S(\alpha) < S(\beta)$, then $\alpha < \beta$;
(II) If $S(\alpha) = S(\beta)$, then
  (a) If $H(\alpha) < H(\beta)$, then $\alpha < \beta$;
  (b) If $H(\alpha) = H(\beta)$, then $\alpha = \beta$.

where $S(\delta) = \mu_\delta - \nu_\delta$ and $H(\delta) = \mu_\delta + \nu_\delta$ are the score and accuracy functions of IFN $\delta = (\mu_\delta, \nu_\delta)$, respectively.

Atanassov and Gargov [13] introduced the interval-valued intuitionistic fuzzy set, which is an effective extension of the IFS. It can be defined as follows:

Definition 3. Let a set X be fixed, an interval-valued intuitionistic fuzzy set (IVIFS) $\tilde{A}$ in X is defined as follows:

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) > | x \in X\}, \tag{4}$$

where $\mu_{\tilde{A}}(x) \subseteq [0, 1]$ and $\nu_{\tilde{A}}(x) \subseteq [0, 1]$ are intervals, and for all $x \in X$, $\sup \mu_{\tilde{A}}(x) + \sup \nu_{\tilde{A}}(x) \le 1$. The pair $\tilde{\alpha} = (\mu_{\tilde{\alpha}}(x), \nu_{\tilde{\alpha}}(x))$ is called an interval-valued intuitionistic fuzzy number (IVIFN) and can be simply denoted as $\tilde{\alpha} = (\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}}) = ([\mu_{\tilde{\alpha}}^L, \mu_{\tilde{\alpha}}^U], [\nu_{\tilde{\alpha}}^L, \nu_{\tilde{\alpha}}^U])$, where $\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}} \subseteq [0, 1]$ and $\mu_{\tilde{\alpha}}^U + \nu_{\tilde{\alpha}}^U \le 1$.

The OWA operator is a widely used tool for aggregating crisp informations. To derive the OWA aggregation over a continuous interval argument, Yager [6] developed the C-OWA operator as follows:

Definition 4. A C-OWA operator is a mapping f: $\Omega \to R$ associated with a basic unit interval monotonic (BUM) function Q, such that:

$$f_Q([a^L, a^U]) = \int_0^1 \frac{dQ(y)}{dy} \cdot [a^U - (a^U - a^L)y] dy, \tag{5}$$