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ASYMPTOTIC INTEGRATION OF SECOND-ORDER IMPULSIVE DIFFERENTIAL EQUATIONS

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Abstract. We initiate a study of the asymptotic integration problem for second-order nonlinear impulsive differential equations. It is shown that there exist solutions asymptotic to solutions of an associated linear homogeneous impulsive differential equation as in the case for equations without impulse effects. We introduce a new constructive method that can easily be applied to similar problems. An illustrative example is also given.

1. Introduction

There is a large body of literature devoted to the asymptotic integration of second-order nonlinear differential equations $x'' = f(t, x)$, see [1] for excellent survey of almost all results up to 2007. In most papers the authors show that for any given real numbers a and b there is a solution $x(t)$ of the nonlinear equation asymptotic to a linear function

$$y = at + b, \quad t \to \infty.$$

This means that the nonlinear equation $x'' = f(t, x)$ has linear like solutions as $t \to \infty$.

To the best of our knowledge there is hardly any work concerning the asymptotic integration of impulsive differential equations. In this work we aim to initiate the study of the asymptotic integration for impulsive differential equations of the form

$$\begin{cases} x'' = f(t, x), & t \neq \theta_i, \quad t \geq 1, \quad i = 1, 2, \dots, \\ \Delta x - p_i x = f_i(x), & t = \theta_i, \\ \Delta x' - q_i x' = \tilde{f}_i(x), & t = \theta_i, \end{cases} \tag{1}$$

where $f, f_i, \tilde{f}_i$ are continuous functions, $\{\theta_i\}$ is a strictly increasing sequence of real numbers such that $\lim_{i\to\infty} \theta_i = \infty$, $\{p_i\}$ and $\{q_i\}$ are sequences of positive real numbers, and Δ is the impulse operator defined by $\Delta u|_{t=\theta_i} = u(\theta_i+) - u(\theta_i-)$, where $u(\theta_i\pm) = \lim_{t\to\theta_i\pm} u(t)$.

The novelty of our work lies in observing that $y = at + b$ is indeed a general solution of an associated homogeneous equation $x'' = 0$, which is obtained from $x'' = f(t, x)$ by setting $f(t, x) \equiv 0$. Indeed, we may write $y = av(t) + bu(t)$, where $u(t) = 1$ and $v(t) = t$ are the principal and nonprincipal solutions of (2), respectively, see [4]. Naturally, we consider the associated linear homogeneous impulsive equation

$$\begin{cases} x'' = 0, & t \neq \theta_i, \quad t \geq 1, \quad i = 1, 2, \dots, \\ \Delta x - p_i x = 0, & t = \theta_i, \\ \Delta x' - q_i x' = 0, & t = \theta_i \end{cases} \tag{2}$$

in place of $x'' = 0$ as in equations without impulses. This observation is a crucial step towards investigating the asymptotic integration problem for differential equations. We will show that for any given real numbers a and b there is a solution of equation (1) asymptotic to piecewise linear function

$$y = ax_1(t) + bx_2(t), \quad t \to \infty,$$

where

$$x_1(t) = t \prod_{i=1}^{\overline{n}(t)} (1 + q_i) + \sum_{i=1}^{\overline{n}(t)} \left[\theta_i (p_i - q_i) \prod_{j=1}^{i-1} (1 + q_j) \prod_{j=i+1}^{\overline{n}(t)} (1 + p_j) \right], \quad x_2(t) = \prod_{i=1}^{\overline{n}(t)} (1 + p_i) \tag{3}$$

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