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On the solution of a class of complex symmetric linear systems

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Abstract. We present an iterative method for solving the complex symmetric system $(W + iT)x = b$, where $T \in \mathbb{R}^{n\times n}$ is a positive definite matrix and $W \in \mathbb{R}^{n\times n}$ is indefinite. Convergence of the method is investigated. The induced preconditioner is applied to accelerate the convergence rate of the GMRES(ℓ) method and the numerical results are compared with those of the Hermitian normal splitting (HNS) preconditioner.

Keywords: complex symmetric linear system, Hermitian normal splitting, skew-normal splitting, modified.

1 Introduction

We consider the system of linear equations

$$Ax \equiv (W + iT)x = b, \tag{1}$$

where $W, T \in \mathbb{R}^{n\times n}$, $b \in \mathbb{C}^n$ and $i = \sqrt{-1}$. We assume that the matrices W and T are symmetric. Systems of the form (1) arise in a variety of scientific and engineering applications. Practical background of this kinds of the problems can be found in [1, 2, 8, 9, 11, 13, 12] and references therein.

Several iterative methods have been presented to solve the system (1) in the literature (for example see [3, 14, 15]). In [3], Bai et al. proposed the Hermitian and skew-Hermitian splitting (HSS) method to solve non-Hermitian positive definite system of linear equations. The HSS iteration method with the Hermitian and skew-Hermitian parts of the matrix A, which are defined by $H = (A + A^H)/2 = W$ and $S = (A - A^H)/2 = iT$, can be directly applied to solve the system (1). However, a modified version of the HSS (MHSS) method was presented by Bai et al. in [1] to solve the system (1) which can be summarized as following.

The MHSS method. Let $x^{(0)} \in \mathbb{C}^n$ be an initial guess. For $k = 0, 1, 2, \ldots$ until the sequence of iterates $\{x^{(k)}\}_{k=1}^{\infty}$ converges, compute the next iterate $x^{(k+1)}$ via the following procedure:

$$\begin{cases} (\alpha I + W)x^{(k+\frac{1}{2})} = (\alpha I - iT)x^{(k)} + b, \\ (\alpha I + T)x^{(k+1)} = (\alpha I + iW)x^{(k+\frac{1}{2})} - ib, \end{cases} \tag{2}$$

where α is a given positive constant and I is the identity matrix.

When both of the matrices W and T are symmetric positive semidefinite with at least one of them (e.g., W) being positive definite, the theoretical analysis shows that the MHSS method converges unconditionally to the unique solution of (1) (see [1]). In each iterate of the MHSS method two subsystems with the coefficient matrices $\alpha I + W$ and $\alpha I + T$ should be solved. Since both of these matrices are symmetric positive definite, the corresponding systems can be solved exactly using the Cholesky factorization or inexactly using the conjugate gradient (CG) method or its preconditioned version, PCG.

When W is symmetric indefinite matrix, then $\alpha I + W$ may be indefinite or singular. In this case, the MHSS iteration method may fail to converge. In [6], Bai designed the skew-normal splitting (SNS) to solve non-Hermitian positive definite systems. The SNS method for solving (1) can be described as following.

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