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Multiple solutions for the fractional Laplacian problems with different asymptotic limits near infinity*

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Abstract In this paper we obtain the existence of two one-sign nontrivial solutions for the fractional Laplacian equations with the nonlinearity having different asymptotic limits at infinity via the mountain pass theorem and the cut-off techniques.
Keywords fractional Laplacian, mountain pass theorem, cut-off technique, different asymptotic limits at infinity.
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1 Introduction

In this paper we consider the existence of nontrivial solutions for the following nonlocal elliptic problem

$$\begin{cases} (-\Delta)^s u = f(x,u) & x \in \Omega, \\ u = 0, & x \in \mathbb{R}^N \setminus \Omega. \end{cases} \tag{1.1}$$

where $s \in (0,1)$ is fixed, Ω is an open bounded subset of $\mathbb{R}^N$ with Lipschitz boundary, $N > 2s$, and $(-\Delta)^s$ is the fractional Laplace operator, which (up to normalization factors) is defined as

$$-(-\Delta)^s u(x) := \int_{\mathbb{R}^N} \frac{u(x+y)+u(x-y)-2u(x)}{|y|^{N+2s}} dy, \quad x \in \mathbb{R}^N. \tag{1.2}$$

Along the paper, we suppose in the equation (1.1) that the nonlinearity $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory mapping which satisfies the following conditions:

(f_1) $f(x,0) \equiv 0$ and $\lim\limits_{t\to 0} \dfrac{f(x,t)}{t} = \alpha$ uniformly for a.e. $x \in \Omega$;

(f_2) there are $\beta_\pm > 0$ such that $\lim_{t\to\pm\infty} \frac{f(x,t)}{t} = \beta_\pm$ uniformly for a.e. $x \in \Omega$.

We note here that condition (f_2) implies that f grows linearly in t at infinity under the situation that the asymptotic limits β_+ and β_- may be different. It follows from (f_2) that the function f verifies the subcritical growth condition

(f) $|f(x,t)| \leqslant a_0(1+|t|^{p-1})$ for a.e. $x \in \Omega$ and all $t \in \mathbb{R}$ $\big(a_0 > 0,\, p \in (1, \frac{2N}{N-2s})\big)$.

The problem (1.1) admits a trivial solution $u = 0$ due to $f(x,0) \equiv 0$. We are interested in the existence of nontrivial weak solutions for the problem (1.1). A weak solution for (1.1) is a function $u : \mathbb{R}^N \to \mathbb{R}$ such that

$$\begin{cases} \displaystyle\int_{\mathbb{R}^{2N}} \frac{(u(x)-u(y))(\varphi(x)-\varphi(y))}{|x-y|^{N+2s}} dxdy = \int_\Omega f(x,u)\varphi dx \quad \forall\, \varphi \in H_0^s(\Omega), \\ u \in H_0^s(\Omega). \end{cases} \tag{1.3}$$

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