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Yutong Chen, Jiabao Su



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Multiple solutions for the fractional Laplacian problems with different asymptotic limits near infinity*

Yutong Chen Jiabao Su

School of Mathematical Sciences, Capital Normal University,
Beijing 100048, People's Republic of China

Abstract In this paper we obtain the existence of two one-sign nontrivial solutions for the fractional Laplacian equations with the nonlinearity having different asymptotic limits at infinity via the mountain pass theorem and the cut-off techniques.

Keywords fractional Laplacian, mountain pass theorem, cut-off technique, different asymptotic limits at infinity.

2010 Mathematics Subject Classification Primary: 35A15, 35A16, 35R09, 35R11, 45K05, 58E05.

1 Introduction

In this paper we consider the existence of nontrivial solutions for the following nonlocal elliptic problem

$$\begin{cases} (-\Delta)^s u = f(x, u) & x \in \Omega, \\ u = 0, & x \in \mathbb{R}^N \setminus \Omega. \end{cases} \quad (1.1)$$

where $s \in (0, 1)$ is fixed, Ω is an open bounded subset of \mathbb{R}^N with Lipschitz boundary, $N > 2s$, and $(-\Delta)^s$ is the fractional Laplace operator, which (up to normalization factors) is defined as

$$-(-\Delta)^s u(x) := \int_{\mathbb{R}^N} \frac{u(x+y) + u(x-y) - 2u(x)}{|y|^{N+2s}} dy, \quad x \in \mathbb{R}^N. \quad (1.2)$$

Along the paper, we suppose in the equation (1.1) that the nonlinearity $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory mapping which satisfies the following conditions:

(f_1) $f(x, 0) \equiv 0$ and $\lim_{t \rightarrow 0} \frac{f(x, t)}{t} = \alpha$ uniformly for a.e. $x \in \Omega$;

(f_2) there are $\beta_{\pm} > 0$ such that $\lim_{t \rightarrow \pm\infty} \frac{f(x, t)}{t} = \beta_{\pm}$ uniformly for a.e. $x \in \Omega$.

We note here that condition (f_2) implies that f grows linearly in t at infinity under the situation that the asymptotic limits β_+ and β_- may be different. It follows from (f_2) that the function f verifies the subcritical growth condition

(f) $|f(x, t)| \leq a_0(1 + |t|^{p-1})$ for a.e. $x \in \Omega$ and all $t \in \mathbb{R}$ ($a_0 > 0$, $p \in (1, \frac{2N}{N-2s})$).

The problem (1.1) admits a trivial solution $u = 0$ due to $f(x, 0) \equiv 0$. We are interested in the existence of nontrivial weak solutions for the problem (1.1). A weak solution for (1.1) is a function $u : \mathbb{R}^N \rightarrow \mathbb{R}$ such that

$$\begin{cases} \int_{\mathbb{R}^{2N}} \frac{(u(x) - u(y))(\varphi(x) - \varphi(y))}{|x - y|^{N+2s}} dx dy = \int_{\Omega} f(x, u) \varphi dx \quad \forall \varphi \in H_0^s(\Omega), \\ u \in H_0^s(\Omega). \end{cases} \quad (1.3)$$

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