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Lower bounds for the blow-up time to a nonlinear viscoelastic wave equation with strong damping

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Abstract

This paper deals with a nonlinear viscoelastic wave equation with strong damping. Under certain conditions on the initial data and the relaxation function, a lower bound for the blow-up time is given by means of a first order differential inequality technique.

Keywords: Lower bound, Blow up, viscoelastic, Strong damping

1. Introduction

In [1], Gazzola and Squassina considered the following initial-boundary value problem:

$$\begin{cases} u_{tt} - \Delta u - \omega \Delta u_t + \mu u_t = |u|^{p-2}u, & \text{in } \Omega \times [0,T], \\ u(x,t) = 0, \quad x \in \partial\Omega, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \end{cases} \tag{1.1}$$

where Ω is an open bounded Lipschitz subset of $\mathbb{R}^n (n \geqslant 1), T > 0, \omega \geqslant 0, \mu > -\omega\lambda_1$, λ_1 being the first eigenvalue of the operator $-\Delta$ under homogeneous Dirichlet boundary conditions. They established global existence of solutions by the potential well method. In addition, not only finite time blow up for solutions starting in the unstable set is proved, but also high energy initial data for which the solution blows up are constructed. Recently, Sun et al. [2] obtained an estimate of the lower bound for the blow-up time by establishing first-order differential inequality when $2 < p \leqslant \frac{2(n-1)}{n-2}$. This work was extended by Guo and Liu [3] to the case when the exponent p lies in the interval $p \in \left(\frac{2(n-1)}{n-2}, \frac{2(n^2-2)}{n(n-2)}\right]$. Further, in the case of $\omega > 0$, by introducing a new auxiliary function Baghaei [4] obtained a lower bound for the blow-up time when $2 < p \leqslant \frac{2n}{n-2}$. This result improves the results obtained in [2, 3].

In presence of the viscoelastic term, the problem has been extensively studied and many results concerning global existence and nonexistence have been proved. For instance, for the equation

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + a|u_t|^{m-2}u_t = b|u|^{p-2}u, \tag{1.2}$$

where $p > 2, m \geqslant 1, a, b > 0$, and g is a positive nonincreasing function, it is well known that solutions with negative initial energy blow up in finite time if $m < p$ and continue to exist

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