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Xiaowei Liu, Jin Zhang

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The transition speed of reaction-diffusion problems with Robin and free boundary conditions ☆

Xiaowei Liu[a,*], Jin Zhang[b,1]

[a] College of Science, Qilu University of Technology, Jinan 250353, China
[b] School of Mathematical Sciences, Shandong Normal University, Jinan 250014, China

Abstract

In [Liu and Lou, J. Differential Equations, 2015], the authors considered the reaction-diffusion equation $u_t = u_{xx} + f(u)$ with Robin and free boundary conditions. For the initial data $\sigma\phi$, there exists $\sigma^* > 0$ such that spreading happens when $\sigma > \sigma^*$ and vanishing happens when $\sigma < \sigma^*$. In the transition case that $\sigma = \sigma^*$, the solution $u(t,x)$ converges to the ground state with a suitable shift: $V(\cdot - \xi(t))$, and $\xi(t)$ tends to a finite number (Case 1) or to ∞ (Case 2) as $t \to \infty$. For both cases, the right free boundary $h(t)$ always propagate to infinity. In this paper, we will discuss the expanding speed of $h(t)$ of these two cases. Actually, $h(t) = \dfrac{1}{\sqrt{-f'(0)}} \ln t + O(1)$ in Case 1 and $h(t) = \dfrac{3}{2\sqrt{-f'(0)}} \ln t + O(1)$ in Case 2.

Keywords: reaction-diffusion equation, free boundary problem, Robin boundary condition, spreading speed

1. Introduction

The nonlinear Stefan problems are widely studied in the recent several years. It was initially proposed by Du and Lin [1] to describe the spreading of a species. By introducing the Stefan boundary condition to the reaction-diffusion models with monostable type nonlinearity, the spreading of species got a better understanding. After that many relevant researches have been developed. For instance, Du, Lou and Zhou gave a rather general theory for the bistable type of nonlinear free boundary problem in [2, 3]. Gu, Lou and Zhou [6] discussed the equation with advection to describe the population dynamics in advective environments. Kaneko and Yamada [7] considered the problem with the Dirichlet boundary condition and gave some sufficient conditions for spreading (i.e., the spreading front of the species moves to infinity and the eventual distribution of the species is a positive stationary state) and vanishing (i.e., the spreading front of the species stays in a bounded interval and the species vanishes eventually). The spreading speed was discussed in [4, 5].

Most of the references mentioned above took the Neumann or Dirichlet boundary condition. Liu and Lou [8] used a Robin boundary condition to describe that the flux of the species invading the habitat from the boundary depends on the density of the boundary. They considered the following problem

$$\begin{cases} u_t = u_{xx} + f(u), & t > 0,\ 0 < x < h(t), \\ u(t,0) = b u_x(t,0), & t > 0, \\ u(t,h(t)) = 0, \quad h'(t) = -u_x(t,h(t)), & t > 0, \\ h(0) = h_0, \quad u(0,x) = u_0(x), & 0 \leqslant x \leqslant h_0 \end{cases} \tag{1}$$

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*Corresponding author: xwliuvivi@hotmail.com.
[1] Email: jinzhangalex@hotmail.com.