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The unique solution of the absolute value equations*

Shi-Liang Wu[†], Cui-Xia Li[‡]
School of Mathematics and Statistics, Anyang Normal University, Anyang, 455000, P.R. China

Abstract

In this paper, we focus on the unique solution of the absolute value equations (AVE). Using an equivalence relation to the linear complementarity problem (LCP), two necessary and sufficient conditions for the unique solution of the AVE are presented. Based on the obtained results, some new sufficient conditions for the unique solution of the AVE are obtained.

Keywords: absolute value equation; unique solution; necessary and sufficient condition
AMS classification: 65F10, 90C05, 90C30

1 Introduction

Consider the absolute value equations (AVE)

$$Ax - |x| = b, \text{ with } A \in \mathbb{R}^{n\times n} \text{ and } b \in \mathbb{R}^n, \tag{1.1}$$

where $|\cdot|$ represents the absolute value. Obviously, the general form of the AVE (1.1) is

$$Ax - B|x| = b, \text{with } A, B \in \mathbb{R}^{n\times n}, \tag{1.2}$$

which was introduced in [1] and investigated in [2]. At present, the AVE (1.1) has attracted considerable attention. This is because the AVE (1.1) occurs in many practical problems of scientific computing and engineering applications, such as linear programs, quadratic programs, linear complementarity problems, interval linear equations, and so on. One can see [3, 4] and the references therein for more detailed descriptions.

To our knowledge, the AVE was first considered in [5] as a useful tool to obtain the numerical solution of the LCP, called the modulus method. To develop the modulus method, its various versions were proposed for solving the LCP, see [6, 7].

In recent years, a large variety of efficient numerical methods for solving the AVE (1.1) have been developed, such as the Picard-HSS method [11], the relaxed nonlinear PHSS-like method [12], Levenberg-Marquardt method [13], the finite succession of linear programs [14], the smoothing Newton method [8], the generalized Newton method [9], the sign accord method [10]. The goal of the above-quoted numerical methods is to find the unique solution of the AVE (1.1).

As is known, it is necessary for us to make sure the existence of the unique solution of the AVE (1.1) before the designing algorithm. So, the research of the unique solution of the AVE (1.1) has been investigated in the literatures. In [4], Mangasarian and Meyer showed that the AVE (1.1) for

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[†]Corresponding author: wushiliang1999@126.com
[‡]lixiatk@126.com