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Energy decay for the wave equation of variable coefficients with acoustic boundary conditions in domains with nonlocally reacting boundary

Tae Gab Ha

Department of Mathematics, and Institute of Pure and Applied Mathematics, Chonbuk National University, Jeonju 54896, Republic of Korea

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ABSTRACT

In this paper, we consider the wave equation of variable coefficients with acoustic boundary conditions in domains with nonlocally reacting boundary. This work is devoted to prove the energy decay for the wave equation of variable coefficients having some boundary reacting conditions. This paper is to improve our previous result in Ha (2016) by applying the variable coefficients case.
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1. Introduction

In this paper, we are concerned with the uniform decay rate of solutions for the wave equation:

$$\begin{cases} u'' - Lu + \rho(u') = 0 & \text{in} \quad \Omega \times (0,+\infty), \\ u = 0 & \text{on} \quad \Gamma_0 \times (0,+\infty), \\ \dfrac{\partial u}{\partial \nu_L} = z' & \text{on} \quad \Gamma_1 \times (0,+\infty), \\ u' + f(x)z'' - c^2 \Delta_\Gamma z + g(x)z' + h(x)z = 0 & \text{on} \quad \Gamma_1 \times (0,+\infty), \\ u(x,0) = u_0(x), \qquad u'(x,0) = u_1(x) & \text{in} \quad \Omega, \\ z(x,0) = z_0(x) & \text{on} \quad \Gamma_1, \end{cases} \tag{1.1}$$

where $Lu = div(A\nabla u) = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right)$ and $\frac{\partial u}{\partial \nu_L} = \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial u}{\partial x_j}\nu_i$. Ω is a bounded domain of $\mathbb{R}^n (n \geq 2)$ with smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$. Here, Γ_0 and Γ_1 are closed and disjoint with $meas(\Gamma_0) > 0$, and $\nu_L = A\nu$, where ν is the outward normal to Γ. $'$ denotes the derivative with respect to time t and Δ_Γ is the Laplace–Beltrami operator.

E-mail address: tgha@jbnu.ac.kr.

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The physical applications of the above system are related to the problem of noise control and suppression in practical applications. The noise sound propagates through some acoustic medium in a room which is characterized by a bounded domain and whose wall, ceiling and floor are described by boundary conditions.

In the case $A = I$, the models with acoustic boundary conditions have been widely investigated (see e.g. [1–5] and a list of references therein). Recently, Frota et al. [6] studied the uniform stability of wave equation with the boundary $(1.1)_3$–$(1.1)_4$, which is called the acoustic boundary conditions to nonlocally reacting boundary. This is a new physical formulation to the acoustic conditions. Since then, the problem with nonlocally reacting boundary has been studied by some authors (cf. [7–9]). More recently, [10] proved the general energy decay estimate for the wave equation with the acoustic boundary conditions to nonlocally reacting boundary.

In general case, the problem that considered the variable-coefficient matrices $A(x)$ with acoustic boundary conditions was studied by a few authors (cf. [11–13]). For example, [14] studied the general decay estimates for the viscoelastic equation with acoustic boundary conditions. There is none, to our knowledge, for the problem with variable coefficients having the acoustic boundary conditions to nonlocally reacting boundary.

Motivated by previous works, the goal of this paper is to improve our previous result in [10] by applying the variable coefficients case. This paper is organized as follows: In Section 2, we give the hypotheses to prove our main result and introduce the energy decay theorem. In Section 3, we prove the general decay rates of (1.1).

2. Preliminaries

We begin this section introducing some notations and our main result. Throughout this paper we define the Hilbert space $\mathcal{H} = \{u \in H^1(\Omega); Lu \in L^2(\Omega)\}$ with the norm $\|u\|_{\mathcal{H}} = \left(\|u\|^2_{H^1(\Omega)} + \|Lu\|^2_{L^2(\Omega)}\right)^{\frac{1}{2}}$. Moreover, $L^p(\Omega)$-norm and $L^p(\Gamma)$-norm are denoted by $\|\cdot\|_p$ and $\|\cdot\|_{p,\Gamma}$, respectively, and $(u,v) = \int_\Omega u(x)v(x)dx$, $(u,v)_\Gamma = \int_\Gamma u(x)v(x)d\Gamma$. Denoting $\gamma_0 : H^1(\Omega) \to H^{\frac{1}{2}}(\Gamma)$ and $\gamma_1 : \mathcal{H} \to H^{-\frac{1}{2}}(\Gamma)$ the trace map of order zero and the Neumann trace map on $\mathcal{H}$, respectively, we have $\gamma_0(u) = u_{|\Gamma}$ and $\gamma_1(u) = (\frac{\partial u}{\partial \nu_L})_{|\Gamma}$ for all $u \in D(\bar{\Omega})$. We denote $W = V \cap H^3(\Omega)$, where $V = \{u \in H^1(\Omega); \gamma_0(u) = 0 \text{ on } \Gamma_0\}$. By Poincaré's inequality, the norm $\|u\|_V = \left(\sum_{i=1}^n \int_\Omega (\frac{\partial u}{\partial x_i})^2 dx\right)^{\frac{1}{2}}$ is equivalent to the usual norm from $H^1(\Omega)$. Since Γ_1 is a compact manifold without boundary, it is possible to use norms in the spaces $H^1(\Gamma_1)$ and $H^2(\Gamma_1)$ by using the tangential gradient and the Laplace–Beltrami operator, respectively. Indeed, we consider the space $H^1(\Gamma_1)$ endowed with the norm $\|z\|^2_{H^1(\Gamma_1)} = \|z\|^2_{2,\Gamma_1} + \|\nabla_T z\|^2_{2,\Gamma_1}$, where ∇_T is the tangential gradient, and $H^2(\Gamma_1)$ equipped with the norm $\|z\|^2_{H^2(\Gamma_1)} = \|z\|^2_{2,\Gamma_1} + \|\Delta_\Gamma z\|^2_{2,\Gamma_1}$.

Now we give the hypotheses for the main result.

($\mathbf{H_1}$) Hypotheses on A.

The matrix $A = \left(a_{ij}(x)\right)$, where $a_{ij} \in C^1(\overline{\Omega})$, is symmetric and there exists a positive constant a_0 such that for all $x \in \overline{\Omega}$ and $\xi = (\xi_1, \ldots, \xi_n)$ we have

$$\sum_{i,j=1}^n a_{ij}(x)\xi_j\xi_i \geq a_0|\xi|^2. \tag{2.1}$$

($\mathbf{H_2}$) Hypotheses on $\boldsymbol{\rho}$.

Let $\rho : \mathbb{R} \to \mathbb{R}$ be a nondecreasing C^1 function such that $\rho(0) = 0$ and suppose that there exists a strictly increasing and odd function β of C^1 class on $[-1,1]$ such that

$$|\beta(s)| \leq |\rho(s)| \leq |\beta^{-1}(s)| \quad \text{ if } \quad |s| \leq 1, \tag{2.2}$$

$$C_1|s| \leq |\rho(s)| \leq C_2|s| \quad \text{ if } \quad |s| > 1, \tag{2.3}$$

where β^{-1} denotes the inverse function of β and C_1, C_2 are positive constants.

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