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Asymptotic profile of solution for the Cauchy problem of beam equation with variable coefficient

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Abstract

We consider the Cauchy problem for the linear beam equation:

$$u_{tt} + u_t + u_{xxxx} - a(t)u_{xx} = 0, \qquad (t,x) \in \mathbb{R}^+ \times \mathbb{R},$$

where $a(t) \sim (1+t)^\alpha$. The purpose of this study is to clarify the behavior of solution depending on the rate α. Here we shall give the asymptotic behavior of the equation in the case $\alpha > -1/2$, by using the method of scaling variables developed by Gallay and Raugel [3].

Keywords: asymptotic profile, beam, variable coefficient, scaling variables
2010 MSC: 35G10, 35B65, 35G25, 35E05

1. Introduction

In this note we study the Cauchy problem for the following one-dimensional linear beam equation with time dependent coefficient:

$$u_{tt} + u_t + u_{xxxx} - a(t)u_{xx} = 0, \qquad (t,x) \in \mathbb{R}^+ \times \mathbb{R}, \tag{1}$$

$$u(0,x) = u_0(x), \qquad u_t(0,x) = u_1(x), \qquad x \in \mathbb{R}. \tag{2}$$

Suppose that the given function $a(t)$ satisfies

$$a(t) \sim (t+1)^\alpha, \qquad a(t) > 0, \qquad |a'(t)| \leq \frac{C_a a(t)^2}{A(t)+1} \tag{3}$$

for some constant C_a with $A(t) := \int_0^t a(\tau)d\tau$. The last assumption in (3) looks intricate but not strange. Indeed, a rough calculation implies $|a'(t)| \leq C(t+1)^{\alpha-1}$, which is compatible with the first assumption. Our interest is directed toward the asymptotic behavior of solution u as $t \to \infty$. In particular we aim at finding the thresholds of α. It is well-known that the solution of the Cauchy problem for damped wave equation: $u_{tt} + u_t - u_{xx} = 0$ behaves as the