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Stability of noninstantaneous impulsive evolution equations ☆

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ABSTRACT

In this paper, we introduce a notation of noninstantaneous impulsive evolution operator, an extension of the classical impulsive evolution operator for linear evolution equations with fixed impulses, which help us to give the concepts of mild solutions to noninstantaneous impulsive Cauchy problems. By characterizing the estimation of noninstantaneous impulsive evolution operator, we establish sufficient conditions to guarantee asymptotic stability of linear and semilinear problems.

1. Introduction

The theory of impulsive differential equations [1,2] arises naturally in physics, engineering, and medical fields. Concerning on the impulsive phenomenon in pharmacotherapy, which starts at any arbitrary fixed points and stays active on a finite time interval, Hernándaz and O'Regan [3] establish a new class of noninstantaneous impulsive evolution equations and present existence and regularity of mild solutions. For more recent new results about impulsive differential models, one can refer to [4–11] and the references therein.

In this paper, we firstly study asymptotic stability of the following linear non-instantaneous impulsive evolution equations

$$\begin{cases} u'(t) = Au(t), \ t \in [s_i, t_{i+1}], \ i \in \mathbb{N}_0 := \{0, 1, 2, \ldots\}, \\ u(t_i^+) = (E + B_i)u(t_i^-), \ i \in \mathbb{N} := \{1, 2, \ldots\}, \\ u(t) = (E + B_i)u(t_i^-), \ t \in (t_i, s_i], \ i \in \mathbb{N}, \\ u(s_i^+) = u(s_i^-), \ i \in \mathbb{N}, \end{cases} \tag{1}$$

where $A : D(A) \subseteq X \to X$ is the generator of a C_0-semigroup $\{T(t) : t \geq 0\}$ on a Banach space X with a norm $\|\cdot\|$, $B_i : X \to X$, $i \in \mathbb{N}$ is bounded linear operator, the sequences $\{t_i\}_{i\in\mathbb{N}_0}$ and $\{s_i\}_{i\in\mathbb{N}_0}$ are satisfied

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with the relation $t_i < s_i < t_{i+1}, i \in \mathbb{N}$ and set $t_0 = s_0 = 0$. Moreover, E denotes the standard identity operator.

Secondly, we study exponential stability of the following semilinear non-instantaneous impulsive evolution equations

$$\begin{cases} u'(t) = Au(t) + f(t, u(t)), \ t \in [s_i, t_{i+1}], \ i \in \mathbb{N}_0, \\ u(t_i^+) = (E + B_i)u(t_i^-) + J_i(u(t_i^-)), \ i \in \mathbb{N}, \\ u(t) = (E + B_i)u(t_i^-) + J_i(u(t_i^-)), \ t \in (t_i, s_i], \ i \in \mathbb{N}, \\ u(s_i^+) = u(s_i^-), \ i \in \mathbb{N}, \end{cases} \tag{2}$$

where $f : [0, \infty) \times X \to X$ and $J_i : X \to X$, $i \in \mathbb{N}$.

We develop the way in [10] and introduce a noninstantaneous impulsive evolution operator $U_T(\cdot, \cdot)$ associated with semigroup operator $T(\cdot)$ and impulsive jump operator B_i (see (3)) without assuming $T(\cdot)B_i = B_iT(\cdot)$, which not only extend the corresponding finite dimensional notation $W(\cdot, \cdot)$ in [10, formula (4)] to infinite dimensional case but also removing the redundant condition $AB_i = B_iA$ in [10]. We also give the concepts of solutions of Cauchy problems for (1) and (2) by virtue of $U_T(\cdot, \cdot)$ respectively. In addition, some sufficient conditions to guarantee asymptotic stability of (1) and asymptotic stability of zero solution and mild solution of (2) are derived.

2. Asymptotic stability of linear problem

Denote $r(t, 0)$ by the number of impulsive points existing in $(0, t)$ and $z^\star = \max\{0, z\}$ for $z \in \mathbb{R}$. We introduce a *non-instantaneous impulsive evolution operator* $U_T(\cdot, \cdot) : [0, \infty) \times [0, \infty) \to X$ given by

$$U_T(t, s) = T((t - s_{r(t,0)})^\star) \left(\prod_{k=r(t,0)}^{r(s,0)+1} (E + B_k)T(t_k - s_{k-1}) \right) T(-(s - s_{r(s,0)})^\star), \tag{3}$$

where we set $\prod_{k=r(t,0)}^{r(s,0)+1} = E$ if $r(s, 0) = r(t, 0)$.

To introduce the concept of mild solution of (1), we consider the set of functions

$$PC([0, \infty), X) = \big\{u : [0, \infty) \to X : u_{|_{J_i}} \in C(J_i, X), J_i = (s_i, t_{i+1}], i \in \mathbb{N}_0 \text{ and } u(t_i^+) \text{ and } u(t_i^-) \text{ exist for each } i \in \mathbb{N}\big\},$$

where $u_{|_{J_i}}$ denotes the domain of u restricted to the subinterval $J_i \subset [0, \infty)$.

Definition 2.1. A function $u(\cdot, s, u_0) \in PC([0, \infty), X)$ is called a mild solution of (1) with $u(s) = u_0$ if

$$u(t, s, u_0) = U_T(t, s)u_0, \quad \forall\ 0 \le s \le t.$$

Note that $T(0) = E$, thus,

$$U_T(t, 0) = T((t - s_{r(t,0)})^\star) \prod_{k=r(t,0)}^{1} (E + B_k)T(t_k - s_{k-1}) \tag{4}$$

and

$$u(t, u_0) := u(t, 0, u_0) = U_T(t, 0)u_0. \tag{5}$$

Definition 2.2. The mild solution $u(\cdot, u_0) \in PC([0, \infty), X)$ is called locally asymptotically stable if $\exists\ \delta > 0$ such that for any $v_0 \in X$ satisfying $\|u_0 - v_0\| < \delta$, $\lim_{t\to\infty} \|u(t, u_0) - u(t, v_0)\| = 0$ holds. The mild solution $u(t, u_0)$ is called globally asymptotically stable if δ can be arbitrary.