Accepted Manuscript

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PII: S0893-9659(17)30211-2
DOI: http://dx.doi.org/10.1016/j.aml.2017.06.009
Reference: AML 5284

To appear in: *Applied Mathematics Letters*

Received date : 2 May 2017
Revised date : 17 June 2017
Accepted date : 17 June 2017

Please cite this article as: X. Wang, Local super-quadratic conditions on homoclinic solutions for a second-order Hamiltonian system, Appl. Math. Lett. (2017), http://dx.doi.org/10.1016/j.aml.2017.06.009

Local super-quadratic conditions on homoclinic solutions for a second-order Hamiltonian system *

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Abstract: In this paper, we study the homoclinic solutions of the following second-order Hamiltonian system

$$\ddot{u} - L(t)u + \nabla W(t,u) = 0,$$

where $t \in \mathbb{R}, u \in \mathbb{R}^N$, $L : \mathbb{R} \to \mathbb{R}^{N\times N}$ and $W : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$. Applying the Mountain Pass Theorem, we prove the existence of nontrivial homoclinic solutions under new weaker conditions. In particular, we use a local super-quadratic condition $\lim_{|x|\to\infty} \frac{W(t,x)}{|x|^2} = \infty$, uniformly in $t \in (a,b)$ for some $-\infty < a < b < +\infty$, instead of the common one $\lim_{|x|\to\infty} \frac{W(t,x)}{|x|^2} = \infty$, uniformly in $t \in \mathbb{R}$, which is essential to show the existence of nontrivial homoclinic solutions for the above system in all existing literature.

Keywords: Homoclinic solution; Hamiltonian system; Local super-quadratic condition.

2000 Mathematics Subject Classification. 34C37; 58E05; 70H05

1 Introduction

Consider the second-order Hamiltonian system

$$\ddot{u} - L(t)u + \nabla W(t,u) = 0, \tag{1.1}$$

where $t \in \mathbb{R}, u \in \mathbb{R}^N$, $L : \mathbb{R} \to \mathbb{R}^{N\times N}$, $W : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ satisfy the following basic assumptions:

(L) $L \in \mathcal{C}\left(\mathbb{R}, \mathbb{R}^{N\times N}\right)$ and $L(t)$ is T-periodic (with $T > 0$) and positive definite;

(W1) $W \in \mathcal{C}^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$, $W(t,0) \equiv 0$, $W(t,x)$ is T-periodic in t;

(W2) $\nabla W(t,x) = o(|x|)$ as $x \to 0$ uniformly for $t \in \mathbb{R}$.

*This work is partially supported by the NNFC (No: 11471278) of China and Key project of Hunan Provincial Education Department (13A093)