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SUPERLINEAR ELLIPTIC SYSTEMS WITH REACTION TERMS INVOLVING PRODUCT OF POWERS

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ABSTRACT. We consider a system of the form

$$\begin{cases} -\Delta u = \lambda g_1(x,u,v) & \text{in} \quad \Omega; \\ -\Delta v = \lambda g_2(x,u,v) & \text{in} \quad \Omega; \\ u = 0 = v & \text{on} \quad \partial\Omega, \end{cases}$$

where $\lambda > 0$ is a parameter, $\Omega \subset \mathbb{R}^N (N \geq 2)$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$ (a bounded open interval if $N = 1$). Here $g_i(x,s,t) : \Omega \times [0,+\infty) \times [0,+\infty) \to \mathbb{R}$ $(i = 1,2)$ are Carathéodory functions that exhibit superlinear growth at infinity involving product of powers of u and v. Using re-scaling argument combined with Leray-Schauder degree theory and a version of Leray-Schauder continuation theorem, we show that the system has a connected set of positive solutions for λ small.

1. INTRODUCTION

In this paper, we study positive solutions of superlinear elliptic systems coupled by reaction terms involving product of powers. For motivation, we consider a system of the form

$$\begin{cases} -\Delta w_1 = w_1^{\alpha_{11}} w_2^{\alpha_{12}} & \text{in} \quad \Omega; \\ -\Delta w_2 = w_1^{\alpha_{21}} w_2^{\alpha_{22}} & \text{in} \quad \Omega; \\ w_1 = 0 = w_2 & \text{on} \quad \partial\Omega\,, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 2)$ with $C^{2,\eta}$ boundary $\partial\Omega$, for some $\eta \in (0,1)$ (a bounded open interval if $N = 1$). The exponents α_{ij} $(i,j = 1,2)$ satisfy the following assumptions:

(A1) $0 \leq \alpha_{11},\ \alpha_{22} < 1$;
(A2) $\alpha_{12}, \alpha_{21} > 0$, $\alpha_{12}\alpha_{21} > (1-\alpha_{11})(1-\alpha_{22})$; and
(A3) $\max\left\{\widehat{\alpha}, \widehat{\beta}\right\} > N-1$, where $\widehat{\alpha} \stackrel{\text{def}}{=} \frac{2(\widehat{p}+1)}{(\widehat{p}\,\widehat{q}-1)_+}$, $\widehat{\beta} \stackrel{\text{def}}{=} \frac{2(\widehat{q}+1)}{(\widehat{p}\,\widehat{q}-1)_+}$ with

$$\widehat{p} \stackrel{\text{def}}{=} \frac{(N+1)\alpha_{12}}{N+1-(N-1)\alpha_{11}} \quad \text{and} \quad \widehat{q} \stackrel{\text{def}}{=} \frac{(N+1)\alpha_{21}}{N+1-(N-1)\alpha_{22}}\,.$$

It was shown in [16, Thm. 1.4 (i)] that under hypotheses (A1)-(A3), any eventual positive *very weak* solution (see [16] for definition) of the system (1.1) has uniform L^∞ estimate and, moreover, the system (1.1) has a positive *strong* solution (w_1, w_2) (to be defined shortly). Note that uniqueness of (w_1, w_2) may not hold in general, even for scalar case, cf. [7]. Let (w_1, w_2) be a positive strong solution of (1.1) and for $\lambda > 0$, define $(z_1, z_2) \stackrel{\text{def}}{=} (\lambda^{-\theta_1} w_1, \lambda^{-\theta_2} w_2)$, where $\theta_1, \theta_2 > 0$ are given by

$$\theta_1 \stackrel{\text{def}}{=} \frac{\alpha_{12}+1-\alpha_{22}}{\alpha_{12}\alpha_{21}-(1-\alpha_{11})(1-\alpha_{22})} > 0 \ \text{and}\ \theta_2 \stackrel{\text{def}}{=} \frac{\alpha_{21}+1-\alpha_{11}}{\alpha_{12}\alpha_{21}-(1-\alpha_{11})(1-\alpha_{22})} > 0\,. \tag{1.2}$$

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Key words: Elliptic systems; superlinear; positive solutions; continuum; bifurcation from infinity.

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