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SUPERLINEAR ELLIPTIC SYSTEMS WITH REACTION TERMS INVOLVING PRODUCT OF POWERS

M. CHHETRI AND P. GIRG

ABSTRACT. We consider a system of the form

$$\begin{cases} -\Delta u = \lambda g_1(x, u, v) & \text{in } \Omega; \\ -\Delta v = \lambda g_2(x, u, v) & \text{in } \Omega; \\ u = 0 = v & \text{on } \partial\Omega, \end{cases}$$

where $\lambda > 0$ is a parameter, $\Omega \subset \mathbb{R}^N (N \geq 2)$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$ (a bounded open interval if $N = 1$). Here $g_i(x, s, t) : \Omega \times [0, +\infty) \times [0, +\infty) \rightarrow \mathbb{R} (i = 1, 2)$ are Carathéodory functions that exhibit superlinear growth at infinity involving product of powers of u and v . Using re-scaling argument combined with Leray-Schauder degree theory and a version of Leray-Schauder continuation theorem, we show that the system has a connected set of positive solutions for λ small.

1. INTRODUCTION

In this paper, we study positive solutions of superlinear elliptic systems coupled by reaction terms involving product of powers. For motivation, we consider a system of the form

$$(1.1) \quad \begin{cases} -\Delta w_1 = w_1^{\alpha_{11}} w_2^{\alpha_{12}} & \text{in } \Omega; \\ -\Delta w_2 = w_1^{\alpha_{21}} w_2^{\alpha_{22}} & \text{in } \Omega; \\ w_1 = 0 = w_2 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 2)$ with $C^{2,\eta}$ boundary $\partial\Omega$, for some $\eta \in (0, 1)$ (a bounded open interval if $N = 1$). The exponents $\alpha_{ij} (i, j = 1, 2)$ satisfy the following assumptions:

(A1) $0 \leq \alpha_{11}, \alpha_{22} < 1$;

(A2) $\alpha_{12}, \alpha_{21} > 0, \alpha_{12}\alpha_{21} > (1 - \alpha_{11})(1 - \alpha_{22})$; and

(A3) $\max\{\hat{\alpha}, \hat{\beta}\} > N - 1$, where $\hat{\alpha} \stackrel{\text{def}}{=} \frac{2(\hat{p}+1)}{(\hat{p}\hat{q}-1)_+}$, $\hat{\beta} \stackrel{\text{def}}{=} \frac{2(\hat{q}+1)}{(\hat{p}\hat{q}-1)_+}$ with

$$\hat{p} \stackrel{\text{def}}{=} \frac{(N+1)\alpha_{12}}{N+1-(N-1)\alpha_{11}} \quad \text{and} \quad \hat{q} \stackrel{\text{def}}{=} \frac{(N+1)\alpha_{21}}{N+1-(N-1)\alpha_{22}}.$$

It was shown in [16, Thm. 1.4 (i)] that under hypotheses (A1)-(A3), any eventual positive *very weak* solution (see [16] for definition) of the system (1.1) has uniform L^∞ estimate and, moreover, the system (1.1) has a positive *strong* solution (w_1, w_2) (to be defined shortly). Note that uniqueness of (w_1, w_2) may not hold in general, even for scalar case, cf. [7]. Let (w_1, w_2) be a positive strong solution of (1.1) and for $\lambda > 0$, define $(z_1, z_2) \stackrel{\text{def}}{=} (\lambda^{-\theta_1} w_1, \lambda^{-\theta_2} w_2)$, where $\theta_1, \theta_2 > 0$ are given by

$$(1.2) \quad \theta_1 \stackrel{\text{def}}{=} \frac{\alpha_{12} + 1 - \alpha_{22}}{\alpha_{12}\alpha_{21} - (1 - \alpha_{11})(1 - \alpha_{22})} > 0 \quad \text{and} \quad \theta_2 \stackrel{\text{def}}{=} \frac{\alpha_{21} + 1 - \alpha_{11}}{\alpha_{12}\alpha_{21} - (1 - \alpha_{11})(1 - \alpha_{22})} > 0.$$

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Key words: Elliptic systems; superlinear; positive solutions; continuum; bifurcation from infinity.

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