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# Robin problems with general potential and double resonance

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**Abstract**

We consider a semilinear elliptic problem with Robin boundary condition and an indefinite and unbounded potential. The reaction term is a Carathéodory function exhibiting linear growth near $\pm\infty$. We assume that double resonance occurs with respect to any positive spectral interval. Using variational tools and critical groups, we show that the problem has a nontrivial smooth solution.

*Keywords:* Indefinite and unbounded potential, double resonance critical groups, regularity theory, Robin boundary condition
*2010 MSC:* 35J20, 35J60, 58E05

## 1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a $C^2$-boundary $\partial\Omega$. We study the semilinear Robin problem:

$$-\Delta u(z) + \xi(z)u(z) = f(z,u(z)) \text{ in } \Omega, \quad \frac{\partial u}{\partial n} + \beta(z)u = 0 \text{ on } \partial\Omega, \tag{1}$$

where $\xi \in L^s(\Omega)$, with $s > N$, is in general indefinite (i.e., sign-changing). The reaction term $f(z,x)$ is a Carathéodory function (i.e., $\forall\, x \in \mathbb{R}$, $z \to f(z,x)$ is measurable and, for a.a. $z \in \Omega$, $x \to f(z,x)$ is continuous). We assume that for a.a. $z \in \Omega$, $f(z,\cdot)$ exhibits linear growth near $\pm\infty$ and asymptotically as $x \to \pm\infty$ the quotient $\frac{f(z,x)}{x}$ stays in any positive spectral interval $[\widehat{\lambda}_m, \widehat{\lambda}_{m+1}]$ (i.e., $\widehat{\lambda}_m > 0$) with possible interaction (resonance) with both endpoints $\widehat{\lambda}_m, \widehat{\lambda}_{m+1}$. So (1) has a "double resonance" feature. In the boundary condition, $\frac{\partial u}{\partial n}$ denotes the normal derivative of $u \in H^1(\Omega)$ defined by extension of the continuous linear map $C^1(\overline{\Omega}) \ni u \to \frac{\partial u}{\partial n} = (\nabla u, n)_{\mathbb{R}^N}$, with $n(\cdot)$ being the outward unit normal on $\partial\Omega$. The boundary coefficient $\beta \in W^{1,\infty}(\partial\Omega)$ and satisfies $\beta(z) \geq 0\, \forall\, z \in \partial\Omega$. If $\beta = 0$, we get the usual Neumann problem. Recently there have been existence and multiplicity results for semilinear elliptic equations with general potential; see [9, 13] (Dirichlet problems), [5, 11] (Neumann problems) and [3, 6, 12, 14] (Robin problems). None of the aforementioned works covers the double resonance situation. We prove the existence of a nontrivial smooth solution, using variational tools together with Morse theory (critical groups).

## 2. Mathematical Background

Let $X$ be a Banach space. By $X^*$ we denote its topological dual and by $\langle\cdot,\cdot\rangle$ the duality brackets for the pair $(X^*, X)$. Given $\varphi \in C^1(X,\mathbb{R})$, we say that $\varphi$ satisfies the "Cerami condition" ("$C$-condition" for short),

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