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A goal–oriented *a posteriori* error estimate for the oscillating single layer integral equation

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Abstract

We construct a reliable goal–oriented *a posteriori* error estimate for the oscillating single layer equation in acoustics. It is based on the computation of the residual corresponding to the primal problem and a reconstruction of the local error for the solution of the dual problem following the Dual-Weighted-Residual approach. We give a numerical example in 2D where the target is the far field value in some direction.

1. Introduction

In many applications, we are interested in the numerical solution of partial differential equations and in the control of the accuracy of this solution. However, for some equations, this solution is not directly usable and some post–processing is required. This is for example the case when dealing with integral equations and the *Boundary Element Method* (BEM).

In this paper, we deal more particularly with integral equations arising from the Helmholtz equation describing the propagation of an acoustic wave

$$\Delta u_{\mathrm{tot}} + k^2 u_{\mathrm{tot}} = 0$$

with u_{tot} the total acoustic field such that $u_{\mathrm{tot}} = u_{\mathrm{i}} + u_{\mathrm{s}}$ where u_{i} is the *incident field* (given) and u_{s} is the *scattered field.* For some scattering object Ω with boundary Γ (possibly open), we add a boundary condition (b.c.) on Γ and a radiation condition at infinity. We suppose here that the propagation domain is two–dimensional and we set $u_{\mathrm{tot}}|_{\Gamma} = 0$ (Dirichlet b.c.).

One integral equation solving the Dirichlet problem is related to the so–called *single layer integral* operator associated with the Helmholtz equation

$$\mathcal{S}_k(\varphi) := \int_{\Gamma} G_k(x, y)\, \varphi(y)\, d\gamma_y = -u_{\mathrm{i}}, \tag{1}$$

$$G_k(x, y) = \frac{i}{4} H_0^{(1)}(k\, |x - y|), \tag{2}$$

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