Nonlinear Analysis: Hybrid Systems 27 (2018) 29–43

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Analysis of a delayed vaccinated SIR epidemic model with temporary immunity and Lévy jumps

Qun Liu [a,b], Daqing Jiang [a,c,d,*], Tasawar Hayat [c,e], Bashir Ahmad [c]

[a] *School of Mathematics and Statistics, Key Laboratory of Applied Statistics of MOE, Northeast Normal University, Changchun, Jilin 130024, PR China*

[b] *School of Mathematics and Statistics, Guangxi Colleges and Universities Key Laboratory of Complex System Optimization and Big Data Processing, Yulin Normal University, Yulin 537000, Guangxi, PR China*

[c] *Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 121589, Saudi Arabia*

[d] *College of Science, China University of Petroleum (East China), Qingdao 266580, Shandong Province, PR China*

[e] *Department of Mathematics, Quaid-i-Azam University, 45320, Islamabad 44000, Pakistan*

HIGHLIGHTS

- A delayed vaccinated SIR epidemic model with temporary immunity and Lévy jumps is studied.
- We establish sufficient conditions for persistence and extinction of the disease.
- We find that a large noise intensity has the effect of suppressing the epidemic to extinction.
- Persistence and extinction have close relationship with Lévy noise and vaccination.

ARTICLE INFO

Article history:
Received 18 January 2016
Accepted 1 August 2017

Keywords:
Stochastic delayed SIR epidemic model
Persistence and extinction
Temporary immunity
Vaccination
Lévy jumps

ABSTRACT

In this paper, we discuss the persistence and extinction of a delayed vaccinated SIR epidemic model with temporary immunity and Lévy jumps. Firstly, we study the existence and uniqueness of the global positive solution with any positive initial value. Then we establish sufficient conditions for persistence and extinction of the disease. Moreover, when the noise is large, we find that a large noise intensity has the effect of suppressing the epidemic, so that it goes to extinction. Results show that the persistence and extinction of the disease have a very closed relationship with the intensity of Lévy noise and the validity period of the vaccination. Some examples and numerical simulations are carried out to show the effectiveness and feasibility of the theoretical results.

1. Introduction

Investigation of epidemic models that incorporate disease-caused death and a varying total population have become one of the most interesting topics in the mathematical theory of epidemiology, largely motivated by the works of Anderson and May (see e.g. [1,2]). Recently, controlling infectious diseases has been an interesting topic in the epidemiology and vaccination has been an important strategy for the elimination of infectious diseases. Consequently, many authors have studied epidemic models with vaccination (see e.g. [3–9]).

* Corresponding author at: School of Mathematics and Statistics, Key Laboratory of Applied Statistics of MOE, Northeast Normal University, Changchun, Jilin 130024, PR China.

E-mail address: jiangdq067@nenu.edu.cn (D. Jiang).

http://dx.doi.org/10.1016/j.nahs.2017.08.002
1751-570X/

On the other hand, many previous models describing the dynamics of diseases by systems of ordinary differential equations without delay. However, delayed differential equations can exhibit much more complex dynamics than differential equations without delay, and stable equilibrium can become unstable with the effects of a time delay (see e.g. [10–14]). Therefore, many scholars have investigated the effects of disease latency or immunity (see e.g. [15–18]).

As a matter of fact, the individuals who have been vaccinated, they could have temporary immunity. Based on this consideration, in this paper, we firstly consider the following deterministic delayed SIR epidemic model with vaccination and temporary immunity

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - \mu S(t) - pS(t) - \beta S(t)I(t) + pS(t-\tau_1)e^{-\mu\tau_1} + \gamma I(t-\tau_2)e^{-\mu\tau_2}, \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - (\mu+\gamma+\alpha)I(t), \\ \dfrac{dR(t)}{dt} = \gamma I(t) + pS(t) - \mu R(t) - pS(t-\tau_1)e^{-\mu\tau_1} - \gamma I(t-\tau_2)e^{-\mu\tau_2}, \end{cases} \tag{1}$$

where S is the number of the individuals susceptible to the disease, I represents the number of the individuals who are infective and R denotes the number of members who are immune to an infection as a result of vaccination, $\frac{dS(t)}{dt}$, $\frac{dI(t)}{dt}$ and $\frac{dR(t)}{dt}$ are the corresponding rates of change with respect to the time t. The parameters have the following meanings: Λ denotes a constant input of new members into the population, μ represents the natural death rate of S, I, R compartments, β denotes the transmission coefficient between compartments S and I, γ denotes the recovery rate of the infective individuals, p is the proportional coefficient of vaccinated for the susceptible, α denotes the disease-caused death rate of infectious individuals, the term $S(t-\tau_1)e^{-\mu\tau_1}$ represents the vaccination and τ_1 represents the validity period of the vaccination, the term $I(t-\tau_2)e^{-\mu\tau_2}$ denotes the temporary immunity and τ_2 is the length of immunity period. The parameters $\Lambda, \mu, p, \beta, \alpha, \gamma$ and τ_i $(i = 1, 2)$ are all positive constants. In system (1), the basic reproduction number [19] is

$$R_0 = \frac{\beta\Lambda}{(\mu+\gamma+\alpha)(\mu+p(1-e^{-\mu\tau_1}))} = \frac{\beta\bar{S}}{\mu+\gamma+\alpha} \tag{2}$$

which determines whether the disease occurs or not, where

$$\bar{S} = \frac{\Lambda}{\mu+p(1-e^{-\mu\tau_1})} \tag{3}$$

is the number of the susceptible individuals at the initial time [19].

However, the deterministic models assume that parameters in the system are all deterministic irrespective of environmental fluctuations, which, from the biological viewpoint, imposes some limitations in mathematical modelling of ecological systems and epidemic models are inevitably affected by the environmental noise (see e.g. [20–23]). Thus, it is necessary to reveal how the environmental noise affects the epidemic model. Using stochastic models can predict the future dynamics of the system accurately. Stochastic differential equation models play an important role in many kinds of branches of applied sciences including infectious dynamics, as they can provide some additional degree of realism compared to their deterministic counterpart (see e.g. [24–27]). Therefore, many authors have considered stochastic biological models and stochastic epidemic models (see e.g. [28–31]).

As we know, population systems may suffer severe environmental perturbations, such as tsunami, volcanoes, avian influenza, SARS, floods, hurricanes, earthquakes, toxic pollutants, etc. These phenomena cannot be described by stochastic continuous models. And so it is feasible to introduce a jump process into the underlying population systems (see e.g. [32–35]). Bao et al. [32,33] did pioneering work in this field. They first investigated two stochastic Lotka–Volterra population systems with Lévy jumps, and studied desired population dynamics of their models. From then on, many results on the epidemic models with Lévy jumps have been reported (see e.g. [36–38]).

Motivated by the above mentioned works, in this paper, we consider the following delayed vaccinated SIR epidemic model with temporary immunity and Lévy jumps for each $t \geq 0$,

$$\begin{cases} dS(t) = (\Lambda - \mu S(t) - pS(t) - \beta S(t)I(t) + pS(t-\tau_1)e^{-\mu\tau_1} + \gamma I(t-\tau_2)e^{-\mu\tau_2})dt \\ \qquad - S(t^-)I(t^-)\left(\sigma dB(t) + \displaystyle\int_{\mathbb{Y}} \gamma(u)\widetilde{N}(dt, du)\right), \\ dI(t) = (\beta S(t)I(t) - (\mu+\gamma+\alpha)I(t))dt + S(t^-)I(t^-)\left(\sigma dB(t) + \displaystyle\int_{\mathbb{Y}} \gamma(u)\widetilde{N}(dt, du)\right), \\ dR(t) = (\gamma I(t) + pS(t) - \mu R(t) - pS(t-\tau_1)e^{-\mu\tau_1} - \gamma I(t-\tau_2)e^{-\mu\tau_2})dt, \end{cases} \tag{4}$$

where $S(t^-), I(t^-)$ are the left limits of $S(t)$ and $I(t)$, respectively, $B = (B(t), t \geq 0)$ is a real-valued Brownian motion with $B(0) = 0$ defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets), $\sigma^2 > 0$ denotes the intensity of the white noise, N is a Poisson counting measure with compensator $\widetilde{N}$ and characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0, \infty)$ satisfying $\lambda(\mathbb{Y}) < \infty$, it is assumed that λ is a Lévy measure such that $\widetilde{N}(dt, du) = N(dt, du) - \lambda(du)dt$, $\gamma : \mathbb{Y} \times \Omega \to \mathbb{R}$ is