Nonlinear Analysis: Hybrid Systems 27 (2018) 213–224

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Synchronization of directed switched complex networks with stochastic link perturbations and mixed time-delays ☆

Yurong Liu [a,d,*], Zidong Wang [b], Lifeng Ma [c], Ying Cui [a], Fuad E. Alsaadi [d]

[a] Department of Mathematics, Yangzhou University, Yangzhou 225002, China
[b] Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom
[c] School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China
[d] Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 3 December 2016
Accepted 28 July 2017

Keywords:
Synchronization
Switched complex networks
General algebraic connectivity
Average dwell time
Link failure

ABSTRACT

In this paper, the synchronization problem is studied for a class of directed switched complex networks. The links among the nodes are perturbed by stochastic noises and the topology varies according to certain predetermined switching rules. The coupled networks under consideration are subject to mixed delays comprising both discrete and distributed ones. A new estimate of the general algebraic connectivity is firstly given for the directed complex networks, based on which the exponential synchronization problem is analyzed by virtue of the average-dwell-time technique. Then, sufficient conditions are derived to guarantee the synchronization in mean square provided that the switching is slow on the average. Subsequently, the switched complex networks with link failures are investigated and it is shown that the synchronization can be achieved if the average link failure ratio does not exceed certain threshold. Finally, a numerical simulation example is presented to demonstrate the effectiveness of the proposed algorithm.

1. Introduction

In the past decade, complex networks have stirred considerable research attention due to their wide applications in various branches of science and engineering including linguistic networks [1], chemical and biological engineering [2,3], social systems [4] and so forth. A complex network consists of a number of coupled nodes whose dynamics are governed by certain differential/difference equations, where the so-called coupling configuration matrix is employed to characterize the connectivity as well as the link strength between nodes. Up to now, there have been a multitude of research results on complex networks concerning the issues of stability, synchronization and state estimation, see e.g. [5–12]. Note that most of the existing results are based on the assumption that the links between nodes are *deterministic* and *fixed*. In real world applications, however, the connection between nodes might evolve with time and also exhibit certain switching/random behaviors due to various reasons such as sudden failures, abrupt environmental changes, sensor aging and dynamical changes of the working conditions. On these occasions, the connectivity may be *stochastic* and *time-varying*, and the resulting complex networks cannot be dealt with along the lines of those in the aforementioned publications.

☆ This work was supported in part the Royal Society of the UK, the National Natural Science Foundation of China under Grants 61329301 and 61374010, the Top Talent Plan of Yangzhou University of China, and the Alexander von Humboldt Foundation of Germany.

* Corresponding author at: Department of Mathematics, Yangzhou University, Yangzhou 225002, China.
E-mail address: yrliu@yzu.edu.cn (Y. Liu).

https://doi.org/10.1016/j.nahs.2017.07.006
1751-570X/

It is worth mentioning that, recently, complex networks with switching topology have been garnering an ever-increasing research interest because of their ability in reflecting the time-varying connection changes. Many results have been reported in the literature regarding the networks coupled with switching topologies, see [13–16] for some latest publications. In particular, the synchronization problems have been investigated in [14,16] where the topologies of the considered networks are switching according to certain predetermined rules. Sufficient conditions have been established by utilizing the average-dwell-time (ADT) method in combination with the linear matrix inequality (LMI) approach. However, to the best of our knowledge, when it comes to the case where the topology is *switching* and subject to additional *stochastic* link noises, the relevant study has been far from adequate despite its practical significance. Note that the stochasticity involved in the switching topology would bring essential difficulties and pose substantial challenges on the analysis of the synchronization issues.

On the other hand, it is well known that time-delays exist in a wide range of practical applications such as chemistry industry, biological engineering and process engineering [12,17–24]. In particular, it is quite often in practice that the signal transmissions are carried out through an amount of parallel channels with a variety of communication constraints, and this gives rise to the distributed delays resulting from the spatial nature of the underlying dynamical systems. A particular feature for distributed delays is that the distant past has less impact as compared to the recent behavior of the state. In the context of complex networks, time-delays are likely to take place in the dynamics of each node, which would inevitably result in considerable complexities in the system analysis and other relevant problems. Generally, the time-delays can be categorized into two types according to the way they occur, i.e., discrete time-delay [25] and distributed time-delay [10]. For complex networks with or without time-delays, several dynamic analysis issues (including synchronization, stability analysis and state estimation) have been extensively investigated, which have led to a plenty of research results reported in the literature [26–29]. However, the corresponding results have been really scattered for complex networks with its nodes connected via links subjected to random perturbations. This is owing mainly to the lack of appropriate techniques to cope with the coupling between mixed time-delays and topological information that is both *stochastic* and *switching*.

Motivated by the above discussions, in this paper, we are concerned with the synchronization problem for a class of switched complex networks subject to stochastic link noises and mixed time-delays. The main contributions of this paper can be highlighted as follows: (1) *a new estimate of the general algebraic connectivity is derived, which plays a paramount role in the establishment of the main results*; (2) *the model of the directed complex network under consideration is quite comprehensive, which takes stochastic link noises, switching topology and mixed time-delays into simultaneous consideration; and* (3) *sufficient conditions are derived for the switched complex networks to achieve the synchronization in the presence of mixed time-delays and stochastic link noises.*

The rest of this paper is arranged as follows. Section 2 formulates the synchronization problem for the directed switched systems subject to stochastic link noises and mixed time-delays. In Section 3, sufficient conditions are derived for the addressed switched complex networks, guaranteeing the exponential synchronization in mean square. In Section 4, an illustrative numerical example is provided to show the effectiveness and usefulness of the proposed approach. Section 5 gives our conclusions.

Notations: The notations are quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I_n is the $n \times n$ identity matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a square matrix, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ means the largest and smallest eigenvalues of A, respectively. $X \otimes Y$ denotes the Kronecker product of matrices X and Y. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the following complex dynamical network consisting of N identical nodes with switched topology and mixed time-delays:

$$\frac{dx_i(t)}{dt} = f(x_i(t)) + g(x_i(t-\tau(t))) + \int_{t-d(t)}^{t} h(x_i(s))ds + c\sum_{l=1}^{N} a_{ij}(t,\sigma(t))(x_j(t) - x_i(t)), \tag{1}$$
$$x_i(t) = \varphi_i(\theta), \quad \theta \in [-r, 0], \quad i = 1, 2, \ldots, N,$$

where $x_i(t) = \left[x_{i1}(t)\, x_{i2}(t)\, \cdots\, x_{in}(t)\right]^T$ denotes the state vector of the ith node; $\sigma(t)$: $[0, +\infty) \to M = \{1, 2, \ldots, m_0\}$ is a switching signal (also called switching rule); $f, g, h : \mathbb{R}^n \to \mathbb{R}^n$ are continuous nonlinear vector-valued functions; $\tau(t)$ and $d(t)$ stand for, respectively, the time-varying discrete and distributed time-delays; $A_{t,\sigma(t)} =: \left(a_{i,j}(t, \sigma(t))\right)$ is the weighted adjacency matrix with $a_{i,j}(t, \sigma(t)) > 0$ if there is a link from node j to node i at time t, otherwise $a_{i,j}(t, \sigma(t)) = 0$; and c represents the coupling coefficient, while $\varphi_i(\cdot)$ stands for the initial condition.

In (1), $r \geq 0$ is a given constant. We assume that there exist positive constants τ_0, τ_m, τ_M and d_M such that $\dot{\tau}(t) \leq \tau_0 < 1$, $0 \leq \tau_m \leq \tau(t) \leq \tau_M \leq r$ and $0 \leq d(t) \leq d_M \leq r$. For nonlinear vector-valued functions, we make the following assumption.