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Gossip based asynchronous and randomized distributed task assignment with guaranteed performance on heterogeneous networks[☆]

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ABSTRACT

The main contribution of this paper is a novel distributed algorithm based on asynchronous and randomized local interactions, i.e., gossip based, for task assignment on heterogeneous networks. We consider a set of tasks with heterogeneous cost to be assigned to a set of nodes with heterogeneous execution speed and interconnected by a network with unknown topology represented by an undirected graph. Our objective is to minimize the execution time of the set of tasks by the networked system. We propose a local interaction rule which allows the nodes of a network to cooperatively assign tasks among themselves with a guaranteed performance with respect to the optimal assignment exploiting a gossip based randomized interaction scheme. We first characterize the convergence properties of the proposed approach, then we propose an edge selection process and a distributed embedded stop criterion to terminate communications, not only task exchanges, while keeping the performance guarantee. Numerical simulations are finally presented to corroborate the theoretical results.

1. Introduction

In this paper we deal with the problem of distributing evenly a set of indivisible tasks over a network of agents who have to process them. To keep the presentation general, we consider tasks with different costs and agents with different execution speeds.[1] This problem, which we called *discrete consensus* in [2], is a generalization of the *quantized consensus* problem proposed by Kashyap et al. in [3] where all tasks (or tokens) have equal weight (or cost) and task execution speeds are not considered, namely they are assumed to be the same for all nodes. We say that the assignment is performed on heterogeneous networks to emphasize the fact that each agent has its own execution speed. Agents are assumed to be interconnected by a bidirectional communication network. In accordance with most of the literature in this area, such a network is assumed to be modeled by an undirected graph as opposed to a directed graph. The quantized consensus problem on directed graphs has been studied by Kai and Ishii in [4].

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[1] In [1] some preliminary results leading to this paper were presented. In this manuscript we provide extended proofs, an improved characterization of the convergence properties, a distributed mechanism to terminate interagent communications, and an extended simulation campaign.

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We investigate the distributed task assignment problem in the framework of gossip algorithms. These algorithms were popularized in the control literature by Boyd et al. [5] who proposed and elegantly characterized a simple, randomized and asynchronous scheme to solve the distributed average problem in a network of sensors. This communication scheme became popular in the development of distributed algorithms for multi-agent systems because it does not require network wide synchronization of the interagent interactions as fundamental assumption for its execution. Among the many results that followed, we mention the work by Ravazzi et al. in [6] which characterized randomized affine dynamics with several applications to multi-agent systems, social networks and sensor networks and the very popular survey on gossip algorithms by Dimakis et al. in [7] and references therein. We also mention applications of gossip algorithms proposed by Riazi et al. to the heterogeneous multi-vehicle routing problem in [8] and to home healthcare scheduling problems [9].

Our goal in this paper is that of minimizing the network execution time of the set of tasks. The assignment is performed according to a novel distributed algorithm based on gossip-like asynchronous local state updates between the nodes. As a result of the proposed local interaction mechanism, the achievement of an optimal task assignment is not guaranteed. However, we are able to prove almost sure convergence in finite time to a bounded set containing optimal solutions. We guarantee that the worst case difference between the optimal value of the execution time and the performance of the proposed algorithm is bounded by a constant which does not depend on the network size.

The quantized consensus algorithm by Kashyap et al. [3] and the discrete consensus algorithm proposed by Franceschelli et al. [2] are based on a local balancing rule to redistribute tasks or tokens among selected pairs of nodes and a so called "swap" rule which updates the state of the nodes by simply swapping their set of tasks or tokens. The swap rule is necessary to avoid blocking configurations. It "shakes" the network state to redistribute the load and allows loads composed by discrete tasks to travel in the network, reaching a situation in which a new balancing may occur. The study of the convergence time of this process depends upon the meeting time of two random walkers in a graph and has received a significant attention in [10–12] among others. When considering heterogeneous execution speed, methods developed for the homogeneous case may fail to converge to the desired convergence set. The approach in [2], which considers heterogeneous execution speeds, suffers the limitation that each pair of *swap domains*, namely connected subgraphs induced by nodes with the same speed, should be connected.

The main contribution of this paper is a novel distributed algorithm which allows to remove the mentioned limitation of [2] and considers arbitrary topologies modeled by connected undirected graphs with nodes of arbitrary speed. Furthermore, we characterize the convergence properties of the algorithm in terms of almost sure convergence to a set of task assignments which well approximates the optimal solution. We prove an absolute performance guarantee by showing that in the worst case scenario the execution time obtained by the proposed algorithm does not differ from the optimal one by more than a constant which does not depend on the network size. Thus, the proposed approach is well suited to address the task assignment problem in large networks. Furthermore, we propose a distributed self-triggered stop criterion to terminate a Poisson edge selection process while keeping the performance guarantee.

Finally, we provide numerical simulations to characterize the expected convergence time of the proposed algorithm in line graphs and random graphs and compare it with the performance of the algorithm in [2]. We conclude the Introduction with a brief overview of the state of the art in this area.

1.1. Literature review

One of the first major contributions to the problem of *quantized consensus* which inspired several works in the following years, was proposed in [3]. It consists in a gossip-based algorithm to steer a set of quantized state variables towards a common value. Other significant contributions, still consisting in randomized gossip algorithms, have been provided in [13]. In these papers state variables in the networks are not considered as indivisible tokens or tasks. More recently, in [4] it was proposed a quantized consensus algorithm that, unlike the previous approaches, applies to networks described by directed graphs.

The issue of providing a characterization of the convergence time of quantized consensus algorithms is investigated in depth in [10,14] and more recently in [11].

A series of contributions in the framework of *discrete consensus* have been recently proposed. Apart from [2] that has already been recalled in the first part of this section, we want to mention [15] where a discrete consensus algorithm in networks affected by execution feasibility constraints has been considered. In [16] tasks of different cost and type with capacity and feasibility constraints are considered. Furthermore, authors impose a constraint on the maximum number of tasks executable by individual nodes. Almost sure convergence to a feasible and time-invariant configuration is proved. However, only preliminary results on the converge time are proposed. In [17] a modification of the quantized consensus algorithm is proposed to solve a load balancing problem with tasks of identical size and nodes with limited capacity.

In [18] a discrete consensus algorithm with improved convergence time with respect to quantized consensus algorithms is proposed for Hamiltonian graphs. In [19] the expected convergence time of discrete and quantized consensus has been improved on arbitrary graphs to $\mathcal{O}(n)d(\mathcal{G})$, where n is the number of nodes and $d(\mathcal{G})$ is the diameter of graph $\mathcal{G}$ representing the network topology.

In [20] the distributed task assignment problem in a network of heterogeneous mobile robots with heterogeneous tasks is investigated. The authors exploit gossip based local optimizations to both assign tasks located in a plane and compute optimized routes for robots. Finally, Chopra and Egerstedt investigated in [21,22] heterogeneity in multi-robot systems as the ability of robots with heterogeneous skill sets to serve spatially and temporally distributed tasks.