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Asymptotic stability analysis and design of nonlinear impulsive control systems

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A R T I C L E I N F O

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A B S T R A C T

In this work, the stability analysis and design problems are studied for a class of nonlinear impulsive control systems. Via the multi-comparison system, we establish a new comparison lemma that ensures asymptotic stability of impulsive differential systems. Based on that, we introduce vector Lyapunov functions to derive some efficient conditions and apply the results into the control design of a chaotic system. It is worth pointing out that the component Lyapunov functions need not be strictly positive definite. Necessary comparison shows that we obtain a larger stable region. Finally, some simulation results are presented to verify the derived conclusions.

1. Introduction

During the last several decades, impulsive control has attracted considerable interest in biology, medicine, engineering and economics. The main idea of impulsive control is to change the states instantaneously at certain instants. Three typical examples are the population control system of a kind of insects with the number of insects and their natural enemies as state variables, a chemical reactor system with the quantities of different chemicals server as the states, and a financial system with two state variables of the amount of money in a market and the saving rates of a central bank [1]. In many cases, impulsive control provides an efficient way in dealing with systems especially which cannot endure continuous control inputs, such as a deep-space spacecraft cannot leave its engine on continuously considering that it has only limited fuel supply [2]. For more practical examples, one can see [3–5] and references therein.

Many researchers have studied impulsive control systems in recent years. Some sufficient conditions are derived in [1] for the impulsive control of a class of nonlinear systems. In [6,7], the authors present some sufficient conditions for the stability of impulsive systems with impulses at fixed times, and then the results are used to design impulsive control for a class of nonlinear systems. Impulsive control method is also applied to the stabilization and synchronization of chaotic systems and complex dynamical networks, see [8–19] for more details.

In this work, we establish a new comparison lemma on asymptotic stability of impulsive differential systems. Based on that, some less conservative stability results are derived for the nonlinear impulsive differential systems. Then we apply the conclusions to the design of impulsive controller for a chaotic system firstly introduced in [20]. To improve or generalize existing results more based on a single comparison system [1,6,21], we derive some conclusions in terms of multi-comparison system. In this case, vector Lyapunov functions should be used and the component Lyapunov functions need not be strictly positive definite. That implies, two or more positive semi-definite Lyapunov functions might be a good

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choice if it is not easy to obtain a single positive definite Lyapunov function under required conditions. Meanwhile, the using of more than one comparison system enhances the flexibility provided by different Lyapunov functions. Furthermore, some comparisons with existing works indicate that we give a larger predicted stable region.

The rest of this paper is organized. In Section 2, we give some definitions and a comparison lemma as preliminaries. In Section 3, we present some efficient conditions to prove the asymptotic stability of impulsive differential systems by multi-comparison system. These results are used to design impulsive controller for a chaotic system in Section 4. In Section 5, some simulation results are presented to show the effectiveness of our proposed method. Finally, this paper is concluded in Section 6.

2. Problem formulation and preliminaries

Consider the impulsive differential system with impulses at fixed times

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & t \neq \tau_k, \\ \triangle x(t) = x(t^+) - x(t^-) = I_k(x), & t = \tau_k, \end{cases} \tag{1}$$

where $x \in R^n$ is the state variable, $f : R_+ \times R^n \to R^n$ is continuous with $f(\cdot, 0) = 0$, $I_k : R^n \to R^n$ is continuous with $I_k(0) = 0$, $\{\tau_k\}(k = 1, 2, \ldots)$ are impulsive instants satisfying $0 \le t_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots$, $\tau_k \to \infty$ as $k \to \infty$. Here we suppose that $x(t^-) = x(t)$.

Notations. Given two vectors $v_1 = [v_{11}, \ldots, v_{1n}]^T$ and $v_2 = [v_{21}, \ldots, v_{2n}]^T$ in R^n, $v_1 \bullet v_2$ is defined as $[v_{11}v_{21}, \ldots, v_{1n}v_{2n}]^T$, and $v_1 \preceq v_2$ ($v_1 \prec v_2$) means $v_{1i} \le v_{2i}$ ($v_{1i} \prec v_{2i}$) for $i = 1, 2, \ldots, n$. $\|\cdot\|$ denotes the Euclidean norm. For a class $\mathcal{K}$ function $\alpha(\cdot)$, we mean a continuous and strictly increasing function with $\alpha(0) = 0$.

Definition 1 ([4]). Let $V : R_+ \times R^n \to R_+$, then V is said to belong to class $\mathcal{V}_0$ if

(i). V is continuous in $(\tau_{k-1}, \tau_k] \times R^n$ and for each $x \in R^n$, $k = 1, 2, \ldots$, $\lim_{(t,y)\to(\tau_k^+,x)} V(t, y) = V(\tau_k^+, x)$ exists;

(ii). V is locally Lipschitzian in x.

Definition 2 ([4]). For $(t, x) \in (\tau_{k-1}, \tau_k] \times R^n$, and a class $\mathcal{V}_0$ function V, we define

$$D^+V(t, x) := \lim_{h\to 0^+} \sup \frac{1}{h}[V(t + h, x + hf(t, x)) - V(t, x)].$$

Definition 3 ([2]). A function $f : R^n \to R^n$ is quasimonotone nondecreasing if $x \preceq y$ and $x_i = y_i$ for some $i \in \{1, 2, \ldots, n\}$, then $f_i(x) \le f_i(y)$ for all $i \in \{1, 2, \ldots, n\}$.

Definition 4 ([2]). Multi-comparison system: Let $V : R_+ \times R^n \to R_+^m$ satisfy $V_i \in \mathcal{V}_0$, $i = 1, 2, \ldots, m$, and

$$\begin{aligned} D^+V(t, x) &\preceq g(t, V(t, x)), \quad t \neq \tau_k, \\ V(t, x + I_k(x)) &\preceq \psi_k(V(t, x)), \quad t = \tau_k, \end{aligned}$$

where $g : R_+ \times R_+^m \to R_+^m$ is continuous in $(\tau_{k-1}, \tau_k] \times R^m$, and for each $p \in R^m$, $k = 1, 2, \ldots$, the limit

$$\lim_{(t,q)\to(\tau_k^+,p)} g(t, q) = g(\tau_k^+, p)$$

exists, $g(t, q)$ is quasimonotone nondecreasing in q, and $\psi_k : R_+^m \to R_+^m$ is quasimonotone nondecreasing. Then the following system

$$\begin{cases} \dot{\omega} = g(t, \omega), & t \neq \tau_k \\ \omega(\tau_k^+) = \psi_k(\omega(\tau_k)) \\ \omega(t_0^+) = \omega_0 \succeq 0 \end{cases} \tag{2}$$

is the multi-comparison system of (1).

We firstly give a useful lemma and then derive a new comparison lemma to prove the asymptotic stability of the impulsive differential system.

Lemma 1. *Let m be a finite positive integer, $V : R_+ \times R^n \to R_+^m$, $V_i \in \mathcal{V}_0$, $K : R_+ \to (0, +\infty)^m$, and assume that*

$$\begin{aligned} &K(t) \bullet D^+V(t, x) + D^+K(t) \bullet V(t, x) \preceq g(t, K(t) \bullet V(t, x)), \quad t \neq \tau_k, \\ &K(\tau_k^+) \bullet V(\tau_k^+, x + I_k(x)) \preceq \psi_k(K(\tau_k) \bullet V(\tau_k, x)), \end{aligned}$$

then $K(t) \bullet V(t, x(t; t_0, x_0)) \preceq r(t; t_0, r_0)$ for $t \ge t_0$ if $K(t_0^+) \bullet V(t_0^+, x_0) \preceq r_0$, where $r(t; t_0, r_0)$ is the maximal solution of system (2) on $[t_0, \infty)$; $D^+K(t) = \lim_{h\to 0^+} \sup \frac{1}{h}[K(t + h) - K(t)]$; $g(t, 0) = 0$ and g is continuous in $(\tau_{k-1}, \tau_k] \times R^n$ for each $x \in R^n$, $k = 1, 2, \ldots$; $\lim_{(t,y)\to(\tau_k^+,x)} g(t, y) = g(\tau_k^+, x)$ exists; ψ_k is continuous and quasimonotone nondecreasing.