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# Mach-Effect thruster model

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## ABSTRACT

The Mach-Effect thruster is a propellantless propulsion concept that has been in development by J.F. Woodward for more than two decades. It consists of a piezo stack that produces mass fluctuations, which in turn can lead to net time-averaged thrusts. So far, thrust predictions had to use an efficiency factor to explain some two orders of magnitude discrepancy between model and observations. Here, a detailed 1D analytical model is presented that takes piezo material parameters and geometry dimensions into account leading to correct thrust predictions in line with experimental measurements. Scaling laws can now be derived to improve thrust range and efficiency. An important difference in this study is that only the mechanical power developed by the piezo stack is considered to be responsible for the mass fluctuations, whereas prior works focused on the electrical energy into the system. This may explain why some previous designs did not work as expected. The good match between this new mathematical formulation and experiments should boost confidence in the Mach effect thruster concept to stimulate further developments.

## 1. Introduction

Propellantless propulsion is a concept, which is traditionally associated with tethers, solar sails or photon rockets. With an on-board power source, such as a nuclear reactor, the photon rocket, which converts energy into radiation and uses radiation pressure to produce thrust, is the only propellantless propulsion that is independent of external sources. This makes it in principle interesting for interstellar travel. However, the thrust $F = P/c^2$ is very small and requires Megawatts to produce milli-Newtons of thrust.

Since the 1990s, James F. Woodward has been developing an alternative approach called Mach-Effect thruster [1–5]. It is based on the well-motivated idea by Sciama [6] that inertia is due to the interaction of mass with the gravitational background from the whole universe. This is in fact one of the interpretations of Mach's principle [7] (“mass out there influences inertia here”), which was a guideline for Einstein to develop his theory of general relativity. Although Einstein's theory is not fully Machian, there are well-known and experimentally verified Mach-type-effects such as frame-dragging [8,9], which can be described by the same weak-field approximation of general relativity as used by Sciama [6]. Woodward used Sciama's result to show that time-changing energy content of a body is causing Machian mass fluctuations that are much larger than one would expect from $E = m\cdot c^2$. Woodward then devised a method to use these mass fluctuations for a novel propulsion scheme: Push the mass when it is heavy and pull it back when it is lighter. This cycle can create a time-averaged net linear impulse in one direction that satisfies the definition of a propellantless thruster. Apart from Woodward's own thrust measurements (e.g. see Ref. [1] for a review), in 2016 Buldrini independently replicated this effect [10]. Recently, it has been shown explicitly that such a scheme does not violate conservation of momentum [11].

Of course, energy must still be spent to vary the mass and accelerate it. The power-to-thrust ratio is an important figure of merit to compare it against photon ($P/F = 3\cdot10^5$ W/mN) and other electric thrusters ($P/F = 20–60$ W/mN). At present, typical experimental values for the Mach-Effect thruster [1] are an order of magnitude better than the photon rocket ($P/F = 3\cdot10^4$ W/mN). Woodward is using Piezo crystals both as capacitors and actuators to oscillate their energy and to push/pull them. Both processes must appear at a proper phase between them to produce thrust.

Unlike a rocket, the thrust for a Mach-Effect thruster is not due to the expulsion of a reaction force. Instead, the anticipated magnitude of mass fluctuation and the thrust that can result from those fluctuations is simply calculated using Newton's 2nd law $F = \Delta m\cdot a$. The important question of course is: “How large is the mass fluctuation?”, to calculate the correct thrust and to benchmark this propulsion scheme against photon rockets.

So far, the predictions and the observed thrust values differ by some orders of magnitudes. It was suggested that this may be due to material efficiencies that were not properly considered [1]. The thrust equation used up to now even predicts a dependence on the frequency to the 6th power, which is not observed (power electronics limitations in tests so far). The only trend that was experimentally verified by Woodward and coworkers is that the (on/off transient) effect seems to scale with the fourth power of the applied voltage to the piezo stack (although only 4 data points have been taken up to now) [12]. We will use the same set of data to compare against our model.

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**Nomenclature**

| | |
|---|---|
| a | Acceleration |
| C | Capacity |
| c | Speed of Light (=3 × $10^8$ m/s) |
| d | Diameter |
| $d_{33}$ | Piezoelectric Constant |
| ρ | Density |
| E | Energy |
| ε | Energy Density |
| $\varepsilon_0$ | Electric Constant (=8.854 × $10^{-12}$ F/m) |
| $\varepsilon_{r33}$ | Relative Permittivity |
| F | Force |
| f | Frequency |
| $f_0$ | Resonance Frequency |
| $\phi_g$ | Gravitational Potential |
| G | Newton's Gravitational Constant (=6.67 × $10^{-11}$ $m^3/kg\text{-}s^2$) |
| g | Gravitational field |
| η | Efficiency |
| I | Current |
| k, $k_p$ | Electromechanical Coupling Coefficient |
| l | Length |
| $M_{33}$ | Electrostrictive Constant |
| m | Mass |
| $N_{PZT,Screw}$ | Number of PZT Discs and Screws |
| ω | Angular Frequency |
| P | Power |
| φ | Phase |
| $Q_m$ | Mechanical Quality Factor |
| t | Time |
| tan δ | Dissipation Factor @ 1 kHz |
| V | Voltage |
| v | Velocity |
| Y | Young's Modulus |

After significant improvements of the experimental techniques, the observed thrusts are in the sub-μN - μN range, which requires micro thrust balances with high resolution. Proper analysis and shielding is necessary to rule out possible artifacts such as thermal effects, outgassing or magnetic interactions as demonstrated by Woodward and coworkers [1,13]. Apart from the need for further testing to consolidate the reality of the effect, the large discrepancy between theory and experimental results persists after some 27 years of development and thus raises doubts if the observed effects are due to mass fluctuations. Even more, the lack of a correct model prohibits the development of scaling laws to amplify the effect beyond any doubt.

The most sophisticated model was recently developed by Rodal [14], who describes the movement of the piezo stack by a set of differential equations with over 200 analytical terms taking material properties into account. His model gives exact predictions; however, he must assume an empirical efficiency factor of 0.4% to match experimental data. Moreover, no analytical scaling laws are given in his paper.

Here, a fully analytical model of the Mach-Effect thruster is presented whose predictions match experimental data and allows the design of optimized thrusters based on mass fluctuations by taking both design and material properties into account. The model gives an important insight into how mass fluctuations appear and why the present design works but other designs failed.

## 2. Mach-Effect thruster design

### 2.1. Fundamentals

The current embodiment of the Mach-Effect thruster consists of a stack of piezo discs that is similar in design to typical actuators using ferroelectric (PZT = Lead Zirconate Titanate) materials, which are sold by many suppliers e.g. for ultrasonic applications. In general, if an electric field is applied across such PZT discs, they expand and contract depending on the field strength and direction of the field. The piezo/PZT stack is made of several discs that are mechanically connected in series but electrically connected in parallel (i.e. all discs have the same electric potential applied between their electrodes). This is achieved by always switching the polarity from disc to disc such that every electrode faces another electrode with the same polarity to avoid electric short circuits. Woodward uses brass electrodes which are glued with epoxy between each disc. The whole assembly is clamped with stainless steel screws between two end caps, a larger one made from brass with threaded holes and a smaller one made from aluminum. The screws are tightened to ensure that the piezo stack is well compressed between the stiff end caps. A schematic sketch as well as an actual thruster is illustrated in Figs. 1 and 2.

Clamping is necessary to generate a force. If no clamping is applied, piezos generate maximum movement but no force. On the other hand, if the stiffness of the clamping is equal to the stiffness of the piezo stack, no movement will occur but maximum force will be generated. This situation applies to both the acoustic applications of PZTs as well as the analytical model developed here. Most actuators choose a clamp stiffness that is well below the piezo stiffness, as it is the case for the present Mach-Effect thruster. The whole assembly is connected with an aluminum bracket on the opposite side of the larger brass cap to the test structure – for measurement purposes, that's a thrust balance. A rubber pad (e.g. Sorbothane) is placed in between this connection to damp out vibration artifacts and to mechanically de-couple high frequency vibrations in the piezo-stack assembly from the balance arm.

### 2.2. Basic concept of getting thrust from a variable mass

Let's assume that the mass of a body $m_0$ can change with a certain angular frequency $\omega$. If we push and pull on this mass with the same frequency, it is easy to see that a net force is generated if both mass oscillation and actuator oscillation are in phase or at a phase of 180° (which then results in a change of the direction of force). We simply assume sinusoidal oscillations and use Newton's 2nd law like

$$\begin{aligned} m(t) &= m_0 \sin(\omega t) \\ x(t) &= x_0 \sin(\omega t + \varphi) \\ a(t) &= \frac{d^2x(t)}{dt^2} = -x_0\omega^2 \sin(\omega t + \varphi) = -a_0 \sin(\omega t + \varphi) \end{aligned} \tag{1}$$

where $m_0$ is the stationary mass, $x_0$ the amplitude of the actuator oscillation and $\varphi$ is the phase between mass and actuator oscillation. We get a non-zero force for a 0° phase and a zero force for a 90° phase by making a time-average over one cycle as

$$\begin{aligned} F_{0^\circ-Phase} &= m(t)a(t) = -m_0\sin(\omega t)a_0\sin(\omega t) = -m_0a_0\sin^2(\omega t) \\ \overline{F}_{0^\circ-Phase} &= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} F_{0^\circ-Phase}dt = -\frac{m_0a_0}{2} \\ F_{90^\circ-Phase} &= m(t)a(t) = -m_0\sin(\omega t)a_0\sin\left(\omega t + \frac{\pi}{2}\right) = -m_0a_0\sin(\omega t)\cos(\omega t) \\ \overline{F}_{90^\circ-Phase} &= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} F_{90^\circ-Phase}dt = 0 \end{aligned} \tag{2}$$

The phase is therefore very important. This basic concept shows that a net time-averaged thrust is possible without using propellants.