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Advanced numerical study of the three-axis magnetic attitude control and determination with uncertainties

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ABSTRACT

Attitude motion of a satellite equipped with magnetic control system is considered. System comprises of three magnetorquers and one three-axis magnetometer. Satellite is stabilized in orbital reference frame using PD controller and extended Kalman filter. Three-axis attitude is analyzed numerically with advanced assumptions: inertia tensor uncertainty, disturbances of unknown nature, magnetometer errors are taken into account. Stabilization and determination accuracy dependence on orbit inclination is studied.

1. Introduction

Magnetic control systems are widely used for satellite attitude stabilization. They are by far the cheapest and are among the most reliable, small and lightweight. The drawbacks are the worst accuracy and even underactuation. Three-axis control is discussed in this paper following our basic results obtained in [1,2] where PD controller viability was shown and recipes for control parameters adjustment were provided. This paper focuses on numerical investigation of satellite attitude under assumptions relevant to real orbital motion. Attitude sensors are restricted to a three-axis magnetometer. Geomagnetic induction vector measurements are processed by extended Kalman filter. Similar system was considered in [3]. Control algorithm was based on computationally consuming SDRE technique, disturbing effects taken into account were different. The main difference lies in a pitch flywheel bias which effectively facilitates satellite stabilization process providing "free" stability for two attitude angles [4].

Magnetometer is a common sensor for satellite attitude determination system [5,6]. The problem of attitude determination with only magnetometer measurements is well studied. Extended Kalman filter was proposed for this problem in [7]. Influence of models parameters and disturbing torque on the accuracy of gravitationally stabilized satellite was estimated. Self-initializing filter guaranteeing convergence with any initial state vector estimate was proposed in [8,9]. Promising two-step Kalman filter was applied in [10]. Magnetic field derivative was determined and used for state vector estimation. Present paper focuses on implementation of the extended Kalman filter with minimal state vector consisting of vector part of quaternion and angular velocity vector. This algorithm is sensitive to geomagnetic induction vector rotation. Filter performance on near equatorial and polar orbits is of special interest; however it was not yet investigated.

Present paper studies the dependence of the attitude determination and stabilization accuracy on orbit inclination. Estimation errors significantly affect attitude stabilization process and vice versa, so the whole magnetic attitude determination and control system behavior is investigated jointly. It is shown that control and Kalman filter parameters tuning allows stabilization accuracy of few to dozens degrees in orbital reference frame. Inertia tensor uncertainty, unaccounted constant and/or Gaussian disturbance and magnetometer bias influence are accounted for.

2. Problem statement

Rigid spacecraft angular motion is considered. The satellite is equipped with three mutually orthogonal magnetorquers and three-axis magnetometer. Magnetorquers can produce any restricted dipole moment. Disturbing torques include gravitational and unknown ones. The latter are represented by constant and/or arbitrary Gaussian values. Inertia tensor knowledge is also erroneous.

Two reference frames are used:

$OX_1X_2x_3$ is the orbital reference frame located at the satellite

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center of mass. OX_3 is directed along the satellite radius-vector, OX_1 is directed along the orbital velocity, OX_2 completes the frame to be right-handed;.

$Ox_1x_2x_3$ is the bound frame described by the principal axes of inertia.

Satellite attitude is represented using Euler angles α, β, γ (rotation sequence 2–3-1), direction cosines matrix $\mathbf{A}$ and its elements a_{ij} (used for analytical study) and quaternion $\Lambda = (\mathbf{q}, \ q_0)$ (used for numerical simulation). Angular velocity may represent either absolute motion ($\boldsymbol{\omega}$ and its components ω_i) or relative motion with respect to orbital reference frame ($\boldsymbol{\Omega}$ and Ω_i). Absolute and relative velocities are related by

$$\boldsymbol{\omega} = \boldsymbol{\Omega} + \mathbf{A}\boldsymbol{\omega}_{orb} \tag{1}$$

where $\boldsymbol{\omega}_{orb} = (0, \ \ \omega_0, \ \ 0)$ is the orbital reference frame angular velocity.

Euler equations for the satellite with arbitrary inertia tensor $\mathbf{J} = \mathrm{diag}(A, B, C)$ are

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M} \tag{2}$$

for absolute angular velocity and

$$\mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega} = \mathbf{M} + \mathbf{M}_{rel} \tag{3}$$

where

$$\mathbf{M}_{rel} = -\mathbf{J}\mathbf{W}_{\boldsymbol{\omega}}\mathbf{A}\boldsymbol{\omega}_{orb} - \boldsymbol{\Omega} \times \mathbf{J}\mathbf{A}\boldsymbol{\omega}_{orb} - \mathbf{A}\boldsymbol{\omega}_{orb} \times \mathbf{J}(\boldsymbol{\Omega} + \mathbf{A}\boldsymbol{\omega}_{orb})$$

for relative angular velocity. $\mathbf{W}_\mathbf{y}$ is a skew-symmetric matrix for any $\mathbf{y}$,

$$\mathbf{W}_\mathbf{y} = \begin{pmatrix} 0 & y_3 & -y_2 \\ -y_3 & 0 & y_1 \\ y_2 & -y_1 & 0 \end{pmatrix}. \tag{4}$$

The torque may contain control part $\mathbf{M}_{ctrl}$ and disturbing part. The latter is divided into gravitational and unknown one, $\mathbf{M} = \mathbf{M}_{ctrl} + \mathbf{M}_{gr} + \mathbf{M}_{dist}$.

Dynamical equations are supplemented with kinematic relations. Quaternion kinematics is

$$\begin{aligned} \dot{\mathbf{q}} &= \tfrac{1}{2}(q_0\boldsymbol{\Omega} + \mathbf{W}_{\boldsymbol{\Omega}}\mathbf{q}), \\ \dot{q}_0 &= -\tfrac{1}{2}\mathbf{q}^T\boldsymbol{\Omega}. \end{aligned} \tag{5}$$

Euler angles are used for analytical analysis, in this case

$$\begin{aligned} \frac{d\alpha}{dt} &= \frac{1}{\cos\beta}(\Omega_2 \cos\gamma - \Omega_3 \sin\gamma), \\ \frac{d\beta}{dt} &= \Omega_2 \sin\gamma + \Omega_3 \cos\gamma, \\ \frac{d\gamma}{dt} &= \Omega_1 - \mathrm{tg}\beta(\Omega_2 \cos\gamma - \Omega_3 \sin\gamma). \end{aligned} \tag{6}$$

Control torque is

$$\mathbf{M}_{ctrl} = \mathbf{m} \times \mathbf{B}$$

where $\mathbf{m}$ is the dipole control moment of the satellite, $\mathbf{B}$ is the geomagnetic induction vector in bound reference frame. Gravitational torque is

$$\mathbf{M}_{gr} = 3\omega_0^2(\mathbf{A}\mathbf{e}_3) \times \mathbf{J}(\mathbf{A}\mathbf{e}_3) \tag{7}$$

where $\mathbf{e}_3 = (0, \ \ 0, \ \ 1)$ is the satellite radius-vector in orbital frame.

Unknown disturbing torque is modelled using three different approaches. Gaussian distribution of the order of $5{\cdot}10^{-7}N{\cdot}m$ allows modelling arbitrary disturbances with small effect on satellite motion since control torque is few orders greater. Constant disturbance on the level of $10^{-7}N{\cdot}m$ augmented with Gaussian one represents more notable disturbance. Constant torque may arise due to aerodynamics or solar pressure acting on a satellite with vast solar panels. The worst case is constant torque of $5{\cdot}10^{-7}N{\cdot}m$ value.

Inclined dipole model is mainly used to represent geomagnetic field. It takes into account three first terms in a Gauss decomposition [11] and allows quite accurate field representation paired with simple computational procedures. Geomagnetic induction vector is

$$\mathbf{B} = \frac{\mu_e}{r^5}(\mathbf{k}r^2 - 3(\mathbf{k}\mathbf{r})\mathbf{r})$$

where $\mathbf{k}$ is the Earth's dipole vector and $\mathbf{r}$ is the satellite radius-vector. Direct dipole model ($\mathbf{k}$ is antiparallel to Earth rotation axis) is used for analytical approaches, geomagnetic induction vector in orbital frame is

$$\mathbf{B}_{orb} = B_0 \begin{pmatrix} \cos u \sin i \\ \cos i \\ -2 \sin u \sin i \end{pmatrix} \tag{8}$$

where $B_0 = \frac{\mu_e}{r^3}$, $\mu_e = 7.812{\cdot}10^6 km^3{\cdot}kg{\cdot}s^{-2}{\cdot}A^{-1}$, r is the satellite radius vector magnitude, u is the argument of latitude, i is the orbit inclination. Geomagnetic induction vector measurements are modelled as

$$\begin{aligned} \tilde{\mathbf{B}} &= \mathbf{A}\mathbf{B}_{orb} + \Delta\mathbf{B} + \boldsymbol{\eta}_\mathbf{B}, \\ \Delta\dot{\mathbf{B}} &= \boldsymbol{\eta}_{\Delta\mathbf{B}} \end{aligned} \tag{9}$$

where $\tilde{\mathbf{B}}$ are the magnetometer readings, $\mathbf{B}_{orb}$ is the modelled induction (inclined field is used in Kalman filter), $\Delta\mathbf{B}$ is magnetometer bias, $\boldsymbol{\eta}_\mathbf{B}$ and $\boldsymbol{\eta}_{\Delta\mathbf{B}}$ are Gaussian magnetometer error and bias rate of change, each with zero mean.

3. Attitude determination using magnetometer

3.1. Kalman filter basics

Kalman filter is a recursive algorithm that uses dynamical system model and sensor readings for actual motion reconstruction. State vector assumption $\hat{\mathbf{x}}_{k-1}^+ = \hat{\mathbf{x}}(t_k)$ is calculated for each discrete time step t_k. Discrete Kalman filter utilizes correction of previous estimate [12]. Consider step $k-1$ along with corresponding state vector estimation $\hat{\mathbf{x}}_{k-1}^+$ and covariance matrix $\mathbf{P}_{k-1}^+$. The goal is to find state vector estimate for the next step $\hat{\mathbf{x}}^+{}_k$. First a priory estimate $\hat{\mathbf{x}}_k^-$ is formed using straight mathematical model integration. It is corrected using sensor measurements vector $\mathbf{z}_k$ to obtain a posteriori estimate $\hat{\mathbf{x}}^+{}_k$. Covariance error matrix $\mathbf{P}_k^-$ is also constructed from the previous step information using Riccati equation. It is then updated to $\mathbf{P}_k^+$ using measurements.

Kalman filter is designed for linear mathematical models and allows the best mean-square state vector estimate. It may be adapted for any non-linear mathematical models of both dynamical system and measurements,

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, t) + \mathbf{G}\mathbf{w}(t), \tag{10}$$

$$\dot{\mathbf{z}}(t) = \mathbf{h}(\mathbf{x}, t) + \mathbf{v}(t) \tag{11}$$

where $\mathbf{w}(t)$ is a Gaussian dynamical model error with covariance matrix $\mathbf{D}$, $\mathbf{G}$ is a matrix of influence of model error on state vector, $\mathbf{v}(t)$ is a Gaussian measurements error with covariance matrix $\mathbf{R}$.

Kalman filter requires right-side functions $\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{h}(\mathbf{x}, t)$ decomposition into the Taylor series in the vicinity of current state vector. Only linear terms are used in the filter. Dynamical system and measurements model matrices are

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}(\mathbf{x}, t)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_k^-, \, t=t_k}, \quad \mathbf{H}_k = \left.\frac{\partial \mathbf{h}(\mathbf{x}, t)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_k^-, \, t=t_k}. \tag{12}$$

Discrete extended Kalman filter uses non-linear dynamical and measurements models for a priory estimate prediction and a posteriori correction [13].

Prediction phase is

$$\begin{aligned} \hat{\mathbf{x}}_k^- &= \int_{t_{k-1}}^{t_k} \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+, t)dt, \\ \mathbf{P}_k^- &= \boldsymbol{\Phi}_k\mathbf{P}_{k-1}^+\boldsymbol{\Phi}_k^\mathrm{T} + \mathbf{Q}_k, \end{aligned} \tag{13}$$

where $\mathbf{Q}_k$ is the covariance matrix of discrete-time process noise, it is