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A nonlinear viscoelastic bend stiffener steady-state formulation

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ABSTRACT

Bend stiffeners are essential components of a flexible riser system, employed to ensure a smooth transition at the upper connection and to protect the riser against over bending and from accumulation of fatigue damage. The highly nonlinear rate dependent behavior of these structures directly affects the integrity assessment of the riser in one of its most critical regions, the top connection. A steady-state formulation (disregarding inertial forces) and numerical solution procedure is developed in this work employing the perturbation method for a nonlinear viscoelastic bend stiffener large deflection beam model subjected to harmonic loading conditions. For stochastic loading conditions, the response is calculated employing the superposition principle by summing up the steady-state result of a number of individual frequency components. A time domain formulation is also derived employing the state-variable approach for the numerical solution of the resulting hereditary integral in the governing equations. A case study is presented for the top connection system of a 4″ ID flexible riser using relaxation and tensile experimental data obtained from a typical class of bend stiffener polyurethane. Harmonic and stochastic input loading conditions are employed for time and frequency domain model comparison/validation and to assess loading history and frequency influence in the curvature response.

1. Introduction

The flexible pipe system is an important part of the overall offshore oil and gas field development, connecting floating production facilities and subsea equipments. Their complex composite pipe wall construction generally leads to low bending stiffness combined with high axial and torsional stiffness, being specifically designed for each application. In the hung-off segment from the supporting offshore platform structure, a bend stiffener is employed to guarantee that the pipe will not over-bend, which may cause unlocking of the pressure armor layer. They are made with polyurethane material usually showing an initial cylindrical part followed by a conical shape and mainly employed for dynamic applications where its design must keep flexible riser service life within acceptable limits (e.g. 20–25 years). According to API Specification 17J [1], the polymer used for bend stiffener manufacture should be resistant to seawater, chemical and ultraviolet exposure and temperature limits for the specified service life, taking into account ageing effects on the mechanical response. Special attention should be given to the interface between the pipe and the bend stiffener, where riser external sheath damage are likely to appear and the large polyurethane structure can also significantly influence the riser heat dissipation.

The modeling approach commonly employed for an efficient design and analysis of the bend stiffener consists of a quasi-static large deflection two dimensional slender beam model, including a short flexible pipe segment and disregarding the gap between the structures. The input loading conditions for this *intermediate quasi-static model* are obtained from stochastic global dynamic analyses, defined in terms of effective top tensions and angle variations. A practice usually adopted for the riser and bend stiffener assessment consists in evaluating the floater motions through time domain realizations of its response spectra, obtained from the vessel's own RAOs (Response Amplitude Operator) and prescribe them to a finite element model of the riser. Another approach is to consider the coupling effects between the floater and the submerged structures (risers and mooring lines), taking into account inertial and damping effects, which becomes increasingly important for deeper waters. Although the riser top connection is mainly dominated by wave frequency components, as pointed out by Caire et al. [2], the coupled approach may allow a more accurate assessment of multi-directional wave energy spreading and consideration of low frequency components.

In order to enable a consistent loading transfer from the three dimensional global dynamic analyses results to an inter-

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mediate or local cross-section model, careful consideration is required. This transposition of results is often less developed and conservative practices are consequently introduced (Smith et al. [3]). An alternative approach for the three dimensional analysis is the use of a quasi-3D methodology, which implies that the time series of the relative angles are processed with an eigenvalue analysis to determine the dominant and the weakest bending planes. Those are then projected in the dominant and weak directions and applied as independent load cases in the top connection model. More information on transposition and directionality has been presented by Grealish et al. [4] and for simplification purposes the two dimensional model will be adopted here.

In terms of polyurethane material modeling, the bend stiffener assessment is traditionally carried out considering elastic response, linear or nonlinear, disregarding its rate dependency characteristics. The time dependence implies that the stress $\sigma(t)$ at time t depends on the preceding strain history, $\varepsilon(\tau)$, $\tau \in (-\infty, t]$, or that the strain $\varepsilon(t)$ at time t depends on the preceding stress history, $\sigma(\tau)$, $\tau \in (-\infty, t]$, as described by Wineman [5]. Theoretically, it means that the whole loading history should be employed for the viscoelastic assessment. In practice, the observed mechanical response and the constitutive equations (including the modified superposition principle employed in this work) are simplified based on the assumption of *fading memory*, i.e. the current stress depends more on recent deformations than past deformations.

The rate dependent intermediate quasi-static model is directly related to the large deflection formulation of viscoelastic beams, which has recently been the subject of some researchers (e.g. Lee [6], Bahraini et al. [7]). The nonlinear viscoelastic behavior assessment of bend stiffeners has been introduced by Caire et al. [8], who performed experimental relaxation and tensile tests with a typical bend stiffener polyurethane, observing the highly nonlinear rate dependent mechanical response of the material. By implementing the modified superposition principle constitutive equation into a commercial finite element package and comparing with the hyperelastic modeling approach, they have demonstrated through a case study the rate dependency importance for system curvature response assessment. The relaxation response observed for the bend stiffener polyurethane at a number of strain levels indicates that the material presents a considerably decay in the first hours of experiment. For a consistent and computationally efficient analysis, it may be convenient to eliminate the initial transient response, without losing the loading history influence, until the system exhibits a steady-state response as time tends to infinity. For a uniform viscoelastic beam subjected to sinusoidal tip loading, a mathematical formulation to obtain the steady-state response has been developed by Vaz and Ariza [9] employing the perturbation theory.

In this work, a steady-state formulation and numerical solution procedure is presented in Section 3.5 for the top connection large deflection beam model with a nonlinear viscoelastic bend stiffener subjected to harmonic loading conditions. The mathematical formulation is developed with the constitutive equation based on the modified superposition principle and employing the perturbation theory to obtain the periodic response after the transient response dies out. Consequently, the quasi-static formulation involves both material and geometrical non-linearities where inertial forces are disregarded. For non-periodic loading conditions and steady-state model validation, a time domain formulation is derived in Section 3.4, employing the state-variable approach for numerical solution of the resulting hereditary integrals. The same experimental data obtained by [8] is adopted for the case study presented in Section 4.

2. Bend stiffener polyurethane behavior

The theory of nonlinear viscoelasticity provides the basis for the development of a constitutive equation that can accurately describe the mechanical response of the bend stiffener polyurethane, which incorporates characteristics of elastic solids and viscous fluids. Although the theory was formulated circa 40 years ago, there is no generally accepted and well-defined form for the nonlinear viscoelastic constitutive equations as there is for linear viscoelasticity, as pointed out by Wineman [5]. In the following Section 2.1 the constitutive equation based on the modified superposition principle is presented, while the response to harmonic oscillation is discussed in 2.2. The experimental data of a typical bend stiffener polyurethane obtained by means of stress relaxation and tensile tests is presented in 2.3 as a function of frequency.

2.1. Nonlinear viscoelastic constitutive equation

Pipkin and Rogers [10] proposed an integral series representation in which the first term is a single integral with a nonlinear integrand. The model is also called the *modified superposition method* and has the following general form,

$$\mathbf{\Pi}(t) = \sum_{n=1}^{\infty} \mathbf{P}_n(t) \tag{1}$$

where $\mathbf{\Pi}$ is the *second Piola-Kirchhoff stress tensor* and $\mathbf{P}_n$ is defined as,

$$\mathbf{P}_n(t) = \frac{1}{n!}\int_{0_-}^{t} \cdots \int_{0_-}^{t} d_{\mathbf{E}(\tau_1)} \ldots d_{\mathbf{E}(\tau_n)} \mathbf{R}[\mathbf{E}(\tau_1), t-\tau_1 ; \ldots ; \mathbf{E}(\tau_n), t-\tau_n] \tag{2}$$

Because the mechanical response depends on the strain history, it is important to distinguish between the current time t and a previous representative time τ. In the equation above, $\mathbf{E}$ is the Green-St Venant strain tensor defined by the following relation with the right Cauchy-Green strain tensor $\mathbf{C}$, $\mathbf{E}(\mathbf{X}, t) = 1/2(\mathbf{C}(\mathbf{X}, t) - \mathbf{I})$ where $\mathbf{X}$ is the reference configuration. At times τ for which the strain tensor is differentiable, $d_{\mathbf{E}(\tau_n)}\mathbf{R}$ can be written as [10],

$$d_{\mathbf{E}(\tau_n)}\mathbf{R}[\mathbf{E}(\tau_n), t-\tau_n] = \frac{\partial \mathbf{R}[\mathbf{E}(\tau_n), t-\tau_n]}{\partial \mathbf{E}} \dot{\mathrm{E}}(\tau_n) d\tau_n \tag{3}$$

The series terminates at the nth term whenever the strain history is an n-step history and can be directly determined by the difference between experimental results for n-step histories and the prediction based on $(n-1)$-step data. The first approximation $(n=1)$ of Eq. (1) is exact for one-step tests and can be completely determined by such tests. For arbitrary histories it is expected to provide good approximations. The same single integral representation can be obtained by redefining the kernel functions of the multiple integral representation, presented by Green and Rivlin [11], as functions of their smaller time arguments (Nolte and Findley [12], Findley et al. [13]). It can be verified that, for the unidimensional case, this leads to the following relation,

$$\sigma(t) = \int_{0_-}^{t} \frac{\partial R[\epsilon(\tau), t-\tau]}{\partial \epsilon(\tau)} \frac{d\epsilon(\tau)}{d\tau} d\tau \tag{4}$$