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Deriving environmental contours from highest density regions

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ABSTRACT

Environmental contours are an established method in probabilistic engineering design, especially in ocean engineering. The contours help engineers to select the environmental states which are appropriate for structural design calculations. Defining an environmental contour means enclosing a region in the variable space which corresponds to a certain return period. However, there are multiple definitions of environmental contours for a given return period as well as different methods to compute a contour. Here, we analyze the established approaches and present a new concept which we call highest density contour (HDC). We define this environmental contour to enclose the highest density region (HDR) of a given probability density. This region occupies the smallest possible volume in the variable space among all regions with the same included probability, which is advantageous for engineering design. We perform the calculations using a numerical grid to discretize the original variable space into a finite number of grid cells. Each cell's probability is estimated and used for numerical integration. The proposed method can be applied to any number of dimensions, i.e. number of different variables in the joint probability model. To put the highest density contour method in context, we compare it to the established inverse first-order reliability method (IFORM) and show that for common probability distributions the two methods yield similarly shaped contours. In multimodal probability distributions, however, where IFORM leads to contours which are difficult to interpret, the presented method still generates clearly defined contours.

1. Introduction

1.1. Purpose of environmental contours

Engineers have to design any marine structure in such a way that it is able to withstand the loads induced by the environment. As the environment, i.e. wind, waves and currents, continually change and cannot be predicted for long periods of time, the environment is often modeled stochastically by defining probability density functions, $f(x_j)$. Then, the structure is designed to withstand all but some extremely rare environmental states, e.g. all waves with significant wave heights, H_s, less than a threshold, h_s, with a cumulative probability or *exceedance probability* of α, i.e. $Pr(H_s \leq h_s) = 1 - \alpha$ or $Pr(H_s > h_s) = \alpha$. In general notation for any random variable, X_1, there exists a threshold, x_1, which fulfills

$$F(x_1) = Pr(X_1 \leq x_1) = \int_{-\infty}^{x_1} f(x)dx = 1 - \alpha. \tag{1}$$

The exceedance probability, α, corresponds to a certain *recurrence* or *return period*, T, which describes the average time period between two consecutive environmental states above the threshold, x_1. The threshold is called *return value*. For example, to comply with standards a marine structure such as an offshore wind turbine is required to withstand significant wave heights, H_s, with a return period, T, of 50 years [20].

Often, however, structural safety depends not only on one variable, but on the occurrence of combinations of p variables, $\{X_j\}_{j=1}^{p}$. When two variables are of importance, e.g. significant wave height, H_s, and spectral peak period, T_p, a joint probability density function can be defined and an environmental contour can be calculated which encloses the subset (or *region*) of environmental states that the structure has to be designed for. Here, we call this region *design*

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Nomenclature

α	Exceedance probability [-]
α_W, β_W, γ_W	Parameters of a Weibull distribution [-]
$\bar{f}$	Cell-averaged joint probability density [-]
$\bar{F}(f_m)$	Probability enclosed by a contour of f_m probability density [-]
$\bar{f}_{X1}$	Cell-averaged probability density in dimension 1 [-]
$\bar{f}_{X2\vert X1}$	Cell-averaged probability density in dimension 2 conditional on x_1 [-]
β	Radius in U-space used in IFORM [-]
$\mathcal{F}$	Failure region [-]
μ_2, σ_2	Parameters of a normal distribution [-]
θ	Angle [deg]
$\tilde{\mu}_{Hs}$, $\tilde{\sigma}_{Hs}$	Parameters of a log-normal distribution [-]
a_1, a_2, a_3, b_1, b_2, b_3	Fitted parameters of the conditional distribution [-]
C	Set making up the environmental contour [-]
$F()$	Cumulative distribution function [-]
$f()$	Probability density function [-]
f_m	Minimum probability density of the enclosed region / constant probability density along the contour [-]
f_m^*	Normalized minimum probability density [-]
H_s	Significant wave height, random variable [m]
h_s	Significant wave height, realization [m]
$H_{s,25}$	25-year return value of the significant wave height based on its marginal distribution [m]
$H_{s\oint}$	Maximum significant wave height along the contour [m]
j	Dimension index [-]
K, L	Number of grid cells in the respective dimension [-]
k, l	Grid cell index [-]
$LN()$	Log-normal distribution [-]
M	Random variable in general variable space [-]
m	Realization of the random variable in general variable space [-]
n	Total number of environmental states in a given time period [-]
$N()$	Normal distribution [-]
p	Number of variables / dimensions [-]
P_f	Failure probability [-]
p_{Hs}	Mixture coefficient [-]
Pr()	Probability function [-]
R	Set enclosed by the environmental contour (highest density region) [-]
r_{MO}	Reference point [-]
T	Return period [years]
T_p	Spectral peak period [s]
T_z	Zero-upcrossing period, random variable [s]
t_z	Zero-upcrossing period, realization [s]
$T_{z\oint}$	Maximum zero-upcrossing period along the contour [s]
U	Random variable in standard normal space [-]
u	Realization of the random variable in standard normal space [-]
X	Random variable in original space [-]
x	Realization of the random variable in original space [-]
z	Number of components [-]
HDC	Highest density contour [-]
HDR	Highest density region [-]
IFORM	Inverse first order reliability method [-]

region (Fig. 1). Often the most critical structural response is associated with very high or low values of environmental variables, i.e. with environmental conditions located at the boundary of the design region. Consequently, standards allow engineers to calculate structural responses for a limited set of environmental *design conditions* along the contour instead of requiring engineering calculations based on a high number of possible variable combinations spread over the complete design region [8]. If there are more than two variables the concept of environmental contours leads to environmental surfaces (3 variables) or environmental manifolds (>3 variables). Here, for simplicity we also refer to these as environmental contours.

1.2. Different definitions and methods

As there are different mathematical definitions for environmental contours one has to further specify which kind of environmental contour is being calculated. Different concepts of environmental contours lead to different design loads and consequently to different structural responses [1]. Originally, environmental contours arose from the concept of return values in univariate probability density functions which are calculated based on one-sided exceedance over threshold (Fig. 2*a*). Consequently, a logical definition for an environmental contour is (i) constant one-sided exceedance in all directions of the p-dimensional variable space, $Pr(X_1 > x_1, X_2 > x_2, \ldots, X_p > x_p) = \alpha$. The bottom panel in Fig. 2*a* shows the contour for the two-dimensional joint distribution of X_1 and X_2. However, for design purposes not only the highest values of a variable can be of interest, but also the lowest. For example, when designing an offshore structure, low values of the peak period, T_p, have to be considered as the structure's natural frequencies can be either higher or lower than the average peak period. Consequently, another possible definition for an environmental contour is (ii) two-sided exceedance over threshold (Fig. 2*b*; e.g. [21]). A third possibility is to define an environmental contour to have (iii) constant probability density, f_m, along its path enclosing the most likely environmental states (Fig. 2*c*). In this case a T-year return period means that on average every T years an environmental state with a probability density less than f_m occurs. In the broader statistics literature the variable region enclosed by such a contour is called a *highest density region* (HDR) [19]. Although HDRs are a logical concept for environmental contours, yet no author has strictly followed this definition. The *design curve* introduced by Haver [14] is a related concept since it is a line of constant probability density, but only one-sided exceedance is considered. The constant probability density approach described by Det Norske Veritas [8] does define a fully closed contour of constant probability density. However, it is designed in such a way that it is unclear how much probability is enclosed by the contour. Instead the contour's probability density, f, is chosen to be the joint probability density of the (x_1, x_2)-variable combination with x_1=return value based on the marginal x_1-distribution and x_2=an associated x_2 value (Fig. 3c). Leira [23], however, has indeed used the HDR definition but only after a transformation of the original

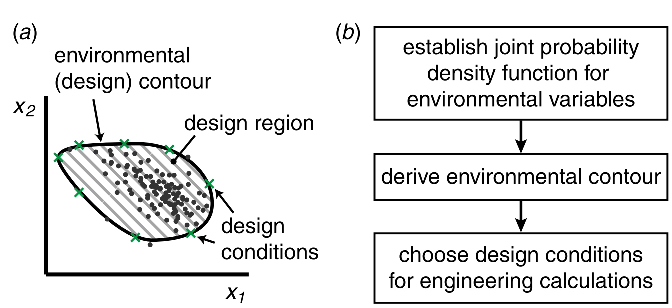

Fig. 1. Concept of an environmental contour. (a) The environmental contour encloses all variable combinations which must be considered in the design process (the design region). (b) Flowchart describing the design process utilizing an environmental contour.