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Neutron multiplication in random media: Reactivity and kinetics parameters

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ABSTRACT

Eigenvalue problems for neutron transport in random geometries are key for many applications, ranging from reactor design to criticality safety. In this work we examine the behaviour of the reactivity and of the kinetics parameters (the effective delayed neutron fraction and the effective neutron generation time) for three-dimensional UOX and MOX assembly configurations where a portion of the fuel pins has been randomly fragmented by using various mixing statistics. For this purpose, we have selected stochastic tessellations of the Poisson, Voronoi and Box type, which provide convenient models for the random partitioning of space, and we have generated an ensemble of assembly realizations; for each geometry realization, criticality calculations have been performed by using the Monte Carlo code TRIPOLI-4®, developed at CEA. We have then examined the evolution of the ensemble-averaged observables of interest as a function of the average chord length of the random geometries, which is roughly proportional to the correlation length of the fuel fragmentation. The methodology proposed in this work is fairly general and could be applied, e.g., to the assessment of re-criticality probability following severe accidents.

1. Introduction

Neutron multiplication in stochastic media has attracted intense research efforts, in view of many relevant applications emerging in reactor physics and criticality safety, such as the design of prismatic and pebble-bed reactors with double heterogeneity fuel (Murata et al., 1996; Liang et al., 2013; Brown and Martin, 2004), the analysis of neutron absorbers with dispersed poison grains (Doub, 1961) or MOX fuels with Pu-rich agglomerates (Yamamoto, 2010), optimal radioactive waste storage (Williams, 2003), and the assessment of the safety margins due to the multiplication factor distribution (Pomraning, 1999; Williams, 2000; Williams and Larsen, 2001), only to name a few.

The formal approach to these problems consists in averaging the critical Boltzmann equation with respect to the random configurations, and then solving the corresponding eigenvalue problem (Pomraning, 1991a,b, 1999). Theoretical progress on such topics is hindered by the great amount of ingenuity required to derive exact results, even in the simplest models and configurations (Pomraning, 1999; Williams, 2000, 2003, 2004; Williams and Larsen, 2001). Perturbation theory can be helpful, but several simplifications are usually required, including mono-energetic (or few-group) transport, isotropic scattering, or diffusion approximation (Pomraning, 1999; Williams and Larsen, 2001; Williams, 2004).

In this respect, Monte Carlo simulation offers a convenient tool for the numerical analysis of eigenvalue problems in stochastic media. For this purpose, two complementary strategies have been proposed (Pomraning, 1991a): the former consists in generating by Monte Carlo methods a collection of realizations of random media (obeying some given distribution, which is supposed to accurately model the system under analysis) and then solving the eigenvalue problem for each realization by using a transport code. The ensemble averages will finally yield the moments, and in principle also the full distribution, of the physical observables of interest, e.g., the multiplication factor or the neutron flux shape. Such reference solutions are exact, since the stochastic nature of the medium is entirely preserved, including the effects on neutron trajectories possibly induced by the spatial correlations. The latter consists in developing effective transport kernels capable of reproducing on-the-fly the 'average' behaviour of neutron displacement accounting for the underlying random heterogeneities, such as in the case of the well-known Chord Length Sampling algorithm (Zimmerman, 1990; Zimmerman and Adams, 1991; Donovan and Danon, 2003; Donovan et al., 2003). In this case, a single eigenvalue calculation with effective jump distribution is sufficient, at the expense of sac-

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rificing the correlations (whose effects must be typically neglected in constructing these algorithms).

Reference solutions, although computationally expensive, are nonetheless of utmost importance for the validation of approximate, albeit much faster methods, and for the verification of exact formulas (Levermore et al., 1986; Adams et al., 1989; Malvagi et al., 1992; Su and Pomraning, 1995; Zuchuat et al., 1994; Larsen and Vasques, 2011; Brantley, 2011; Donovan and Danon, 2003; Donovan et al., 2003; Brantley and Palmer, 2009; Brantley, 2009). Significant advances have been made in the numerical simulation of eigenvalue problems in the presence of stochastic inclusions, where objects (typically spheres) are randomly placed within a background matrix (Murata et al., 1996; Liang et al., 2013; Brown and Martin, 2004). Stochastic inclusions emerge for instance in the modelling of the double heterogeneities in prismatic or pebble-bed reactors. In particular, highly sophisticated algorithms have been devised in order to properly take into account boundary effects due to spheres not entirely contained in the medium (Griesheimer et al., 2010; Ji and Martin, 2011). Eigenvalue calculations in stochastic tessellations, where the medium is supposed to be partitioned into a collection of random (fissile and non-fissile) volumes obeying a given mixing statistics (Pomraning, 1991a), have been limited so far to one-dimensional configurations of the rod or slab type (Pomraning, 1999; Williams and Larsen, 2001; Williams, 2004). Such models might represent, e.g., the accidental positioning of fuel lumps into moderating material, in the context of criticality safety, or the displacement of control plates in damaged reactor cores.

In a series of recent papers, we have examined the statistical properties of linear particle transport through d-dimensional stochastic tessellations without multiplication (Lepage et al., 2011; Larmier et al., 2017a). In particular, we have computed reference solutions for the ensemble-averaged scalar flux and the reflection and transmission probabilities for Poisson (Markov) mixing statistics by revisiting the benchmark originally proposed by Adam, Larsen and Pomraning (Adams et al., 1989; Brantley, 2011; Brantley and Palmer, 2009; Brantley, 2009), and we have then extended our findings to the case of Voronoi and Box statistics (Larmier et al., 2017b).

In this work, we will adopt stochastic tessellations (Santaló, 1976; Torquato, 2013) in order to assess the impact of the three-dimensional random fragmentation of fuel elements on the key safety parameters for criticality calculations, including the multiplication factor k_{eff}, the effective delayed neutron fraction β_{eff}, and the effective neutron generation time ℓ_{eff}. In-pile fuel degradation might result from partial core melt-down during severe accidents, with melting, re-solidification and relocation (Hagen and Hofmann, 1987; Hofmann, 1999), as occurred in the case of the Three Mile Island unit 2 (Broughton et al., 1989). The effects of such scenario on neutron kinetics are of utmost importance for the evaluation of the re-criticality probability. To our best knowledge, the influence of random geometries on kinetics parameters has never been addressed before. Starting from a reference UOX or MOX assembly with 17×17 fuel pins, we will consider three perturbed configurations having the central pin, 7×7 central pins and the whole 17×17 pins being randomly fragmented. We will assume that the random re-arrangement after melt-down can be described by ternary mixing statistics (Pomraning, 1991a), accounting for the dispersion of the fuel, the cladding and the moderator, the average linear size of the chunks for each material being a free parameter of the model. For each realization, we will perform criticality calculations by using the Monte Carlo transport code TRIPOLI-4® developed at CEA (Brun et al., 2015), so as to investigate the distribution of k_{eff}, β_{eff} and ℓ_{eff} as a function of the model parameters, including the material compositions, the kind of stochastic tessellation, the linear size of the random chunks, and the number of fragmented fuel pins.

In order to better grasp the physical behaviour of these systems without being hindered by the complexity of all the ingredients involved in nuclear accidents, the analysis of assembly configurations that will be carried out in this work is admittedly highly simplified with respect to the realistic description of fuel degradation: for instance, we will focus exclusively on neutron transport, and we will not include the effects due to thermal-hydraulics, thermo-mechanics or the complex physical-chemical reactions occurring in accidental transients (Hagen and Hofmann, 1987; Hofmann, 1999). Nonetheless, the methodology proposed in this paper is fairly broad and can be applied without any particular restrictions to more sophisticated models.

This paper is organized as follows: in Section 2 we will briefly introduce the stochastic tessellations that will be used in this work. Then, in Section 3 we will describe the benchmark model for the partially melted fuel assembly. Simulation results for the multiplication factor and the kinetics parameters will be presented and discussed in Section 4. Conclusions will be finally drawn in Section 5.

2. Stochastic tessellations

Stochastic tessellations form a convenient class of probabilistic models to partition a given d-dimensional domain into random polyhedral cells (Santaló, 1976; Chiu et al., 2013; Torquato, 2013), and as such have been adopted in a broad spectrum of applications, ranging from material science (Gilbert, 1962; Meijering, 1953) to stereology (Serra, 1982; Santaló, 1976): for an extensive review, see, e.g., (Santaló, 1976; Miles, 1972; Torquato, 2013). In this section, we introduce the three mixing statistics that will be used in order to generate the random geometries for the criticality calculations. In particular, we will recall the algorithms for their construction based on Monte Carlo methods, and their main statistical features. These algorithms have already been discussed elsewhere, but they are reported here in order for the paper to be self-contained. For the sake of conciseness, the three stochastic tessellations will be distinguished by their label m, namely, $m = \mathcal{P}$ for Poisson tessellations, $m = \mathcal{V}$ for Voronoi tessellations, and $m = \mathcal{B}$ for Box tessellations. The algorithms described in the following have been implemented into a computer code that can perform the statistical analysis of an ensemble of realizations, and can produce input files for the TRIPOLI-4® Monte Carlo transport code (Brun et al., 2015).

2.1. Isotropic Poisson tessellations

Markovian mixing is generated by using isotropic Poisson geometries, which form a prototype process of stochastic tessellations: a domain included in a d-dimensional space is partitioned by randomly generated $(d-1)$-dimensional hyper-planes drawn from an underlying Poisson process (Santaló, 1976; Miles, 1970, 1972). Poisson tessellations represent an idealized mathematical model for disordered media, whose prominent advantage lies in the fact that they require a minimal information content, since the associated correlation function is exponential (see comments in Section 2.4). An algorithm has been recently proposed for finite d-dimensional geometries (Serra, 1982; Ambos and Mikhailov, 2011), allowing for the explicit construction of three-dimensional homogeneous and isotropic Poisson tessellations. In the following we will present the algorithm for the construction of these geometries restricted to a cubic box (further details are provided in (Larmier et al., 2016)).