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Modeling neutron count distribution in a subcritical core by stochastic differential equations

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ABSTRACT

Reactor noise, caused both by the probabilistic nature of the fission chains and external reactivity noises, is one of the basic topics in nuclear science and engineering, both in theory and practice. Classical approaches to modeling this noise and neutron count distribution in the detection system rely on the stochastic transport equation for the probability generating function and on transfer function response to random perturbations. In recent years, a third modeling approach has been proposed, relying on *Ito stochastic differential equations*, which enjoys the tractability that the first aforementioned approach has, and at the same time accounts for fluctuations, by modeling noise in terms of Brownian motion. This paper develops the latter approach to incorporate the stochasticity in the detection process to the model equations. The resulting neutron count distributions are explicitly computable.

As an application of our approach we present a straightforward derivation of the well-known Feynman-Y formula. We then propose an alternative to the traditional sampling scheme of this formula, based on *mean absolute deviation*, known from the statistics literature to be more robust than the *mean square deviation* estimator. The study focuses on a single energy point model and neglects the effect of the delayed neutrons. Extensions of the approach to multiple energy levels and the incorporation of delayed neutrons are discussed, as well as further applications of the approach and its advantages over existing diffusion scale approximations.

1. Introduction

Reactor noise and neutron flux fluctuation is one of the basic topics in nuclear science and engineering, both in theory and practice. Applications of the theory of neutron fluctuation may be found both in monitoring and measurements (Uhrig, 1970), and in non destructive assay of special nuclear materials (Ensslin et al., 1998). Fluctuations in the neutron population size may be attributed to two types of statistical noises: Internal noises, governed by the statistical nature of the neutron interactions, and external noises, reflecting stochasticity of other elements of the system, such as temperature fluctuations, mechanical instabilities, electronic noise in the monitoring system and more (Williams, 1974).

Reactor noise and *neutron fluctuations* are general terms used to describe the modeling and sampling of higher moments of the neutron population distribution in multiplying systems due to both internal and external factors. The theory of reactor noise is very different between the so called *Zero Power Reactors* (ZPR), where the main contribution to the statistical variance is due to the internal factors, and full scale power reactors, where the statistical variance is largely dominated by the external factors. Due to the many differences between the internal and external noises, the two are traditionally modeled differently and analyzed using distinct mathematical tools. Internal noises are studied through the probability generating function and the Kolmogorov equation (a comprehensive overview may be found in Pazsit and Pal (2008)), when external noises are often treated as random perturbation on the point reactor kinetics equation (PKE), and are typically analyzed through transfer function input/output analysis (Williams, 1974). In particular, whereas the perturbation is considered random, the analysis does not consider the noise as a true stochastic process, and the results are typically specified in terms of the output response to a fixed perturbation. Indeed, the transfer function is very useful in determining the amplitude of the response to a random noise but says little on the probabilistic characteristics of the response.

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Hayes and Allen (2005) proposed a new modeling approach for reactor noise based on the diffusion approximations,[1] by which the *central limit theorem* (CLT) provides a model for the stochastic fluctuations. The model is expressed in terms of *Ito stochastic differential equations* (SDE) (Karatzas and Shreve, 1991). In a sense, this modeling scheme is intermediate, lying between the deterministic point reactor kinetic equation and the full stochastic transport equation. Formally it corresponds to versions of the point reactor kinetic equation with terms accounting for stochastic perturbations.[2]

The goal of this paper is to develop the SDE approach by incorporating stochasticity associated with the detection process. Despite the crucial role played by the stochastic detection process, existing SDE models have accounted only for the stochastic fluctuations in the neutron population. However, a model that lacks the detection aspect is arguably incomplete, since it is only possible to infer the population size through the detector response. Thus coupling the detector response with the equation for the population size appears to be of utmost importance for any practical implementation of the approach.

This paper offers three main contributions, that are all related to the detection stochasticity. (i) The incorporation of the detection process into the SDE model, in a way that accounts for its stochastic nature. The model is formulated as a *coupled pair* of SDE, cosisting of an equation for the neutron population and another for the detection count. (ii) Derivation of explicit formula for the Feynman variance-to-mean ratio (which also serves as a strong validation of the proposed model). (iii) Based on the proposed model, a discussion of an alternative method for sampling the Feynman-Y curve, via the mean average deviation.

The analysis in this paper is restricted to the single energy point model (an assumption that is also in force in Hayes and Allen (2005)), and the delayed neutrons are neglected (incorporating the delayed neutrons is doable, but extremely lengthy, and in the present context, with no real gain).

The paper is organized as follows. The remainder of the present section describes the motivation. In Section 2 some background on both reactor noise and stochastic analysis is given. Section 3, which constitutes the main contribution, introduces the SDE of relevance, and provides some basic analysis thereof. Section 4 is devoted to a proposed method of sampling the Feynman-Y function, that is theoretically based on our SDE model, for which we provide experimental evidence. The main idea of the method is to use the mean absolute deviation to estimate the variance, rather than the traditional mean squares. Section 5 lists our conclusions from this study.

1.1. Motivation

Introduced in the early 1950's in the seminal work of Feynman (1945) and having been covered by numerous textbooks on the subject since then, the modeling and analysis of reactor noise are important both in theory (Malinovitch and Dubi, 2015; Demeshko et al., 2016) and applications (Diniz and dos Sontas, 2006; Diniz and dos Sontas, 2002). Still, it is widely accepted that reactor noise is not fully understood. In his book from 1974, M. M.R. Williams states (Williams, 1974):

"*…noise analysis of power reactors is in its infancy due mainly to a lack of knowledge about the variety of noise mechanisms involved…In terms of the input - output concept we are not only ignorant of the nature of the input but in many cases of the system response function as well…the great number of noise sources and fluctuating parameters is the main stumbling-block for scientists in the field, especially for those who have become accustomed to the fascinating clearness of zero-power noise studies. At the same time it is precisely the difficulties that constitute the merit of the topic;…*"

Since 1974, the topic has been vastly studied by many contributors, but no fundamental breakthrough was achieved in our basic understanding of how a random fluctuation in the reactor parameters would propagate on to the power level and the neutron flux. Since the SDE model was introduced in Hayes and Allen (2005), the model was adopted by many contributors, including the following (to state a few): in Ha and Kim (2010), the model was extended to a stochastic PDE, allowing 1D spatial dependence of the neutron population, in Ha and Kim (2011), the reactor transient behavior was studied, in Allen (2013), the doubling time of a subcritical assembly was studied and in da Silva (2016), numeric solutions to the SDE were studied.

As already mentioned, in all previous work, the detection process was completely neglected. From the theoretical viewpoint of studying the population dynamics per se, this can be justified: the effect of the detection can merely be thought of as absorption. However, from a practical point of view, modeling the detection process is crucial, since the neutron detections are the only observable that can be directly linked to the neutron population.

2. Background

2.1. The point reactor kinetics equation

The Point reactor Kinetics Equation (PKE), describing the average neutron population, takes the form Ott and Neuhold (1985)

$$\frac{dN(t)}{dt} = \frac{\rho - \beta_{\text{eff}}}{\Lambda} N(t) + \sum_{j=1}^{\kappa} \lambda_j C_j(t) + S(t), \tag{1}$$

$$\frac{dC_j(t)}{dt} = -\lambda_j C_j(t) + \frac{\beta_j}{\Lambda} N(t), \tag{2}$$

where

N denotes the number of neutrons,

C_j the concentration of the jth delayed neutron group precursor,

ρ the reactivity,

Λ the generation time,

λ_j the decay constant of the jth delayed neutron group precursor,

β_j the fraction (in units of reactivity) of the jth delayed neutron group precursor,

β_{eff} the delayed neutron fraction (in units of reactivity), defined by $\beta_{\text{eff}} = \sum_{j=1}^{\kappa} \beta_j$.

This is one of the most basic equations in nuclear engineering, with numerous applications. In common practice we assume that there are $\kappa = 6$ delayed neutron groups, and the values of the parameters $\{\lambda_j\}_{j=1}^{6}, \{\beta_j\}_{j=1}^{6}$ may be found in the literature.

A simplified version of the PKE is the prompt reactivity model, where the delayed neutron fraction is treated as an external source, and the dynamics are governed by a simplified, point equation

$$\frac{dN(t)}{dt} = \frac{\rho - \beta_{\text{eff}}}{\Lambda} N(t) + S(t). \tag{3}$$

[1] It should be clarified that the use of the term *diffusion approximation* in this paper is different than its more standard use in the nuclear physics literature. That is, it refers to the identification of CLT-scale limits describing the neutron dynamics over time (as well as other stochastic processes), not to be confused with *spatial diffusion approximation*, associated with Fick's law, that is often used in relation to the Boltzmann equation.

[2] Diffusion scale approximations have been also used before in this field in the context of partial differential equations (PDE), specifically by appealing to the Fokker-Planck equation (Williams, 1974) (Ch. 5.6).