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# Mathematical model of fracture complexity indicator in multistage hydraulic fracturing

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The multistage hydraulic fracturing has been considered as the major stimulation method used for enhancing well productivity in unconventional low-permeability reservoirs. A definite positive relationship has been showed between fracture complexity and productivity. However, the fracture complexity is currently unknown due to the fact that it cannot be obtained through any existing diagnostic techniques or numerical simulation. In this paper, the pseudo number of fractures is defined as the fracture complexity indicator to describe the complexity of fracture network during multistage hydraulic fracturing. A model for predicting the fracture complexity indicator is established based on the principle of energy balance and minimum energy. Sensitivity analysis of this analytical model demonstrate that the fracture complexity indicator increases with injection time, injection rate and elastic modulus, while decreases with the height of fractures. Case study is performed to check the validibility of this analytical model in four different wells. It indicates that the SRV obtained from micro-seismic date cannot be used as a general criterion for evaluating the effectiveness of fracturing, since larger SRV does not mean higher complexity of fracture network or higher production. However, fracture complexity indicator described in this paper can illustrate the fracture complexity, which is a parameter to evaluate the effectiveness of multistage hydraulic fracturing. This study provides a general method to optimize the fracturing design so as to increase fracture complexity and evaluate the effectiveness of multistage hydraulic fracturing.


## 1. Introduction

The multistage hydraulic fracturing has been considered as the major stimulation method used to enhance oil and gas productivity in unconventional low permeability reservoirs (Waters et al., 2009; Y. Cheng, 2012; Wu and Olson, 2013). This technique includes massive fracturing, simultaneous fracturing, zipper fracturing and modified-zipper fracturing. A large stimulated reservoir volume (SRV) and a complex fracture network are created due to the interaction between hydraulic and natural fractures (Baihly et al., 2007; Lee Matthews and Schein, 2007; Hyunil and Baker, 2012). A definite positive relationship has been presented between fracture complexity and production (Fisher et al., 2002, 2004; Maxwe et al., 2002; Fisher et al., 2004; Guillermo et al., 2015).

During multistage hydraulic fracturing, the process of stress change should be considered during analyzing the mechanical of fracture interaction. The changing stress can significantly reduce horizontal-stress contrast to create transverse fractures and facilitate the opening of natural fractures (Roussel and Sharma, 2011; Morrill and Miskimins, 2015; Ziarani et al., 2014). Therefore, Fracture network can be further complicated by existing natural fractures in reservoir (Gale et al., 2007; Olson, 2008; Wu et al., 2012; Kresse et al., 2013). Previous study including experiments and numerical simulations have explained that the hydraulic fracture can intersect and induce the propagation of existing natural fractures. Since the data obtained from experiments are very limited, it is better to diagnose the complex fracture network through numerical method, which provides a critical link between fracture network and the optimization of stimulation treatments. Therefore, many numerical fracture propagation models have been developed to simulate the complex fracture network propagation in formation with natural fractures, such as the hydraulic fracture network (HNF) model (Xu et al., 2009), the discrete-fracture-network (DFN) model (Meyer and Bazan, 2011; Weng, 2015), the pseudo-3D fracture propagation model (Olson and Dahi-Taleghani, 2009; Weng et al., 2011), the extended-finite-element-method (XFEM) model

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(Dahi-taleghani and Olson, 2011; Keshavarzi et al., 2012), the simplified 3D-Dimensional displacement Discontinuity method (DDM) model (Wu and Olson, 2015a; 2015b, 2016), etc. Several key physical mechanisms of complex fracture network are considered in those numerical fracture propagation models, such as stress-shadow effects, fluid-rate distribution among multiple fractures, interaction of hydraulic fracture and natural fractures, proppant transport, etc.

In order to accurately simulate the process of fractures propagating, a precise description for the existing natural fracture network should be provided to the numerical fracture propagation models. However, the information about the existing natural fracture network is almost impossible to obtain, such as the density and the length of the natural fractures. Consequently, all the numerical fracture propagation models are simulated based on the assumed distribution of natural fracture networks. The natural fractures density can be obtained from core samples analysis, but the use of the core-scale natural fractures density for the entire reservoir is debatable (Manchanda and Sharma, 2014). Moreover, we cannot figure out whether the fractures are the original or created by stress releasing. Micro-seismic mapping is widely used to determine the SRV. However, it does not provide any detail of the fracture structure and density (Maeryhofer et al., 2010; Chong et al., 2014).

More complex fracture network will result in better well productivity. Optimizing the fracture treatments and the amount of fracturing materials (fluid and proppant) can increase the fracture complexity. However, the fracture complexity cannot be obtained by any fracture diagnostic techniques or numerical simulation techniques. Therefore, other methods are desired to characterize the hydraulic fracture complexity and reveal the its relationship to different fracturing parameters, such as injection time, amount of fracturing materials. This work defines the pseudo number of fractures as the fracture complexity indicator to characterize fracture complexity. A mathematical model of fracture complexity indicator in multistage hydraulic fracturing is established according to the principle of energy balance and minimum energy. The sensitivity analysis and case study are then carried out to analyze the main factors of fracture complexity indicator and its relationship to well productivity.

## 2. Model of fracture complexity indicator

### 2.1. The definition of fracture complexity indicator

The fracture initiation and propagation is mainly dominated by the distribution of in-situ stress in conventional fracturing. The fracture will extend along the direction of maximum stress according to the mechanical mechanism. The bi-wing-fracture will be generated in the reservoir, as shown in Fig. 1(a). However, the induced stress created by higher net pressure in multistage hydraulic fracturing will reduce the stress contrast and even change the direction of maximum stress, which will generate new fractures deviating from the maximum horizontal in-situ stress in unconventional low-permeability reservoir. At the same time, the hydraulic fracture can intersect and induce the propagation of existing natural fractures. Therefore, the complex-fracture network will be induced in multistage hydraulic fracturing, as shown in Fig. 1(b). Many studies have already explained the forming mechanisms and conditions of the complex fracture network. In addition, the mapped events from micro-seismic data also illustrate the outline of fracture network in multistage hydraulic fracturing is not the bi-wing sharp. Actually, it is more likely a sharp of ellipsoid, as shown in Fig. 1(c).

Maximizing fracture complexity is one of the main goals of the stimulation design. It plays a more important role in unconventional reservoirs. Many multistage hydraulic fracturing treatments have been used to increase fracture complexity. During the implement of multistate hydraulic fracturing treatment, it is difficult to predict the detailed configuration of hydraulic fracture because of the unknown complicated rock heterogeneities, existing natural fractures, in situ and induced stress distribution. However, in light of the hydraulic fracturing is a process of releasing pressure energy to the rock media and obeying conservation of energy, it is possible to identify fracture complexity depending on the energy balance during fracturing in rock media.

According to the mechanical mechanism, the fracture will propagate in the direction of maximum horizontal in-situ stress firstly, then deviate from the maximum horizontal in-situ stress due to natural fractures and induced stress. With the development of fracture, more and more fractures will appear. It is shown that the fracture complexity increases with the number of fractures, meaning that larger number of fractures could demonstrate a more complexity of fracture network. Even though it is impossible to predict the exact number of fractures and their configurations, fracture complexity can be characterized on the basis of average length of fractures and pseudo number of fractures. The pseudo number of fractures is defined as the fracture complexity indicator to characterize the fracture complexity. The product of the average length of fractures and the pseudo number of fractures is equal to the total length of fractures.

### 2.2. Analysis of energy balance during fracturing

The hydraulic fracturing is a process of releasing pressure energy to the rock media, which sticks to the principle of conservation of energy. To describe the restrictions of hydraulic fracturing, we need to declare the following assumptions:

1) The rock is homogeneous and isotropic in the lateral extension;
2) Rock deforms in its elastic region;
3) Fracture branches are in rectangular shape in vertical plane;
4) Fluid leak-off is negligible.

The total energy injected into fractures is expressed as:

$$E_{inj} = p_{inj} q_{inj} t \tag{1}$$

where $E_{inj}$ *is* energy injected into the fractures, J; $p_{inj}$ is bottom hole injection pressure, Pa; $q_{inj}$ is fluid injection rate,m$^3$/s;$t$ *is* injection time,s.

The total injected energy takes the following forms inside the fractures:

1) pressure energy (potential energy of fluid);
2) kinetic energy of the flowing fluid;
3) strain energy (potential energy of rock);
4) surface energy (energy due to the interfacial tension between solid and fluid phase);
5) heat energy due to the friction of fluid flow.

The pressure energy is denoted by

$$E_p = pV = pnwhx \tag{2}$$

where $E_p$ is pressure energy, J; $p$ is the average pressure in the fractures, Pa; $V$ is total volume of the fractures, m$^3$; $n$ is the pseudo number of fractures (fracture complexity indicator);$w$ is the average width of fractures, m; $h$ is the average height of fractures, m; $x$ is the average length of fractures, m.

The kinetic energy takes the form of