Ultrasonics 82 (2018) 209–216

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Unveiling the polarization of the multimode acoustic fields

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ARTICLE INFO

Article history:
Received 24 January 2017
Received in revised form 27 June 2017
Accepted 30 August 2017
Available online 1 September 2017

Keywords:
SAW
BAW
Interference
Polarization
Sensors
NDT

ABSTRACT

The spatial three dimensional variation of the polarization ellipse characterizing an elastic movement of the solid particles in the harmonically oscillating ultrasonic fields is studied. Such a variation comes into sight because of the interference of various elastic modes generated during the excitation and/or scattering of acoustic waves by the inhomogeneities located on the solid surface. In order to confirm this effect, an innovative method based on finite element approach is used showing a rule to find the form and orientation of the elastic polarization ellipse in general case. Polarization plane may have an arbitrary orientation in anisotropic crystals. The dispersive parameters of the spatially changeable ellipse of polarization in a substrate with perturbed boundary conditions depend on the polarization and on the relative complex-valued amplitudes of the partial elastic modes which contribute differently to the sum of acoustic fields at every point of analysis. Results of a detailed investigation of the mentioned effects in an interdigital transducer formed by aluminum electrodes on the ST-X cut of quartz substrate are presented.

1. Introduction

The wide-spread finite element analysis (FEA) is used nowadays to investigate with high accuracy the ultrasonic wave phenomena and functional characteristics of various distributed systems based on surface (SAW) and pseudo surface (PSAW) acoustic waves (e.g., [1–8]). Nevertheless, to the best of the authors' knowledge, till now nobody paid proper attention to the elastic wave polarization during evaluation of the scattering characteristics concerning the reflection of acoustic waves from the surface inhomogeneities located on a crystal substrate in the most general case.

When talking about the wave reflection, it is necessary to compare with each other only the modes with the certain configuration in space. Otherwise, one has to study barely the energy of the scattered/transmitted fields without a specific consideration of their content [2]. Therefore, it was impossible to talk rigorously about, e.g., the phase of reflection coefficient which is a very important characteristic in many applications relating to the manufacturing of the wide spread SAW and PSAW acoustoelecric devices [5].

At the same time, many investigators have studied the polarization of various plane wave modes propagating in diverse media (e.g., [9–16]). That fact allows one to improve an understanding of diverse wave phenomena in solids aiming at their optimal utilization in a variety of applications (e.g., in geophysical researches [9] or providing the non-destructive examination of the solid surface [15,16]). It was indicated that the sensitivity of Rayleigh wave [15] and Lamb wave [16] polarization to a state of pre-stress or defects may lead to novel structural diagnostic tools.

Earlier no one talked about the wave polarization in a domain where many eigenmodes appeared simultaneously. Accounting for these features by means of the FEA-based calculations allows one to evaluate the contribution of partial acoustic components to the common elastic field. It is necessary, in particular, for an accurate evaluation of the key phenomenological parameters (KPP) of distributed systems, such as the transduction, reflection, and attenuation coefficients per wavelength there. Note that they are commonly known as COM parameters base on the popular Coupling-of-Modes analysis [17]. The latter is valid under rather small perturbations of the surface boundary conditions. However, when these perturbations have an arbitrary strength, a reference to the abbreviation KPP looks to be more adequate manner in the framework of general study [18].

Knowledge of those parameters can reduce dramatically the calculation time and clarify the physical essence of various wave phenomena in distributed piezoacoustic systems. Analytical models, based on the KPP usage, are the excellent alternatives to the direct application of precise, but cumbersome finite element

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http://dx.doi.org/10.1016/j.ultras.2017.08.011
0041-624X/

method to the modeling of numerous acoustoelectric devices [17,18]. At this point the FEA capabilities should be used only as the auxiliary tools to search for the KPP-values.

The present paper investigates the polarization problem for the multimode elastic fields of the ST-X cut of quartz substrate.

2. Evaluation of the harmonic field polarization

Harmonic waves always generate the elliptical polarizations (a particular case of which is a linear polarization). Previously this affirmation was proved only for electromagnetic waves where the vectors of the electric and magnetic fields move over the ellipses belonging to a transverse plane which are perpendicular to the wavenumber vector [19]. In general case an ellipse plane may have an arbitrary spatial orientation and axial ratio which can be found simply using an algorithm [18] described below.

Commonly, an elastic displacement vector $\boldsymbol{u}$ provides all three projections (components) on the Cartesian axes of the working coordinate system (CS): $u_\nu = |u_\nu| \cdot \exp(i\psi_\nu)$ where ν = 1, 2, 3. Omitting the time dependence factor $\exp(i\omega t)$, where $\omega = 2\pi f$ is the angular frequency, let us define this vector as $\boldsymbol{u} = \boldsymbol{U} \cdot \exp(i\psi_3)$, where

$$\boldsymbol{U} = |U_1| \cdot \exp(i\phi_1) \cdot \boldsymbol{x}_0 + |U_2| \cdot \exp(i\phi_2) \cdot \boldsymbol{y}_0 + |U_3| \cdot \boldsymbol{z}_0, \tag{1}$$

$\phi_{1,2} = \psi_{1,2} - \psi_3$, while $\boldsymbol{x}_0$, $\boldsymbol{y}_0$ and $\boldsymbol{z}_0$ mean the unit vectors of the Cartesian basis chosen initially.

One can show that a vector $\boldsymbol{U}$ can be represented - by means of three, in general, rotations of the working CS – as a superposition of its projections on only two main axes of the polarization ellipse:

$$CS = \{X, Y, Z\} \rightarrow CS' = \{X', Y', Z' = Z\} \rightarrow CS'' = \{X'' = X', Y'', Z''\}$$
$$\rightarrow CS''' = \{X''', Y''' = Y'', Z'''\}$$

The first rotation, by angle α around the Z axis, described as

$$\begin{cases} U'_1 = U_1\cos(\alpha) + U_2\sin(\alpha) \\ U'_2 = U_2\cos(\alpha) - U_1\sin(\alpha) \end{cases}, \tag{2}$$

eliminates the imaginary component of the projection of U-vector on the Y'-axis ($\mathrm{Im}(U'_2) = 0$) if

$$\alpha = \tan^{-1}(|U_2|\sin(\phi_2)/(|U_1|\sin(\phi_1))) \tag{3}$$

The second turn around the X'-axis under

$$\beta = \tan^{-1}(-U'_2/|U_3|) \tag{4}$$

allows one to determine the polarization plane of the elastic displacements(X', Z''), eliminating the component U''_2 on the whole $(U''_2 = 0; U''_3 = |U_3|/\cos(\beta))$.

Then, turning the CS'' around Y'' -axis by the angleθ, one can find the projections U'''_1 andU'''_3 of the displacement vector on the main ellipse axes: $U = |U'''_1| \cdot \exp(i\phi'''_1) \cdot x'''_0 + |U'''_3| \cdot \exp(i\phi'''_3) \cdot z'''_0$,where

$$\begin{cases} U'''_1 = U'_1 \cdot \cos(\theta) - U''_3 \cdot \sin(\theta) \\ U'''_3 = U''_3 \cdot \cos(\theta) + U'_1 \cdot \sin(\theta) \end{cases} \tag{5}$$

Denoting $\widetilde{A} = |U''_3/U'_1|$ &$\widetilde{\varphi} = \arg(U''_3/U'_1)$ and using the condition $Re(U'''_3/U'''_1) = 0$ the required θ -value is found to provide the quadrature phase shift between U'''_3 and U'''_1 components so that the equalities $|U| = \sqrt{|U'''_1|^2 + |U'''_3|^2}$ and $\cos(\phi'''_1 - \phi'''_3) = 0$ hold:

$$\theta = \frac{1}{2} \cdot \tan^{-1}(2\tilde{A} \cdot \cos(\tilde{\phi})/(\tilde{A}^2 - 1)) \tag{6}$$

At this time the chosen solid particle moves clockwise (when looking from the $+Y''$ direction) if $\phi'''_1 - \phi'''_3 = \pi/2$, and anticlockwise if $\phi'''_1 - \phi'''_3 = -\pi/2$.

In the absence of dissipation, every acoustic eigenmode has its own phase velocity and elastic polarization as constant during propagation of a plane wave along the chosen direction in a uniform solid. For example, using the traditional calculation technique [20] one can find the normalized components of elastic displacement of the Rayleigh SAW propagating along X-axis of ST-cut of quartz, characterized by the Euler angles (0, 132.75, 0): $u_1 \approx -i \cdot 0.5479$, $u_2 \approx 0.07951$, $u_3 \approx 0.8328$. The corresponding parameters of elastic polarization are as follows:

$$\alpha = \alpha_0 = 0, \quad \beta = \beta_0 \approx -5.45^\circ, \quad \gamma = \gamma_0 = 0, \quad \text{and}$$
$$|U'''_3/U'''_1|_0 \approx 1.527 \tag{7}$$

However, the wave interaction with any material discontinuity may generate (with different efficiency) many types of eigenmodes which propagate in all possible directions. As a result, the polarization ellipse of the total harmonic acoustic field may have an arbitrary form which varies in space depending on the phase shift and amplitude relation between different elastic modes at the chosen points of consideration.

A notability of such effect depends on symmetry of crystals which allows excitation of different acoustic modes simultaneously. This circumstance can be illustrated, first of all, by a simple example of the interaction of acoustic modes in an isotropic solid.

3. On interference of acoustic modes in isotropic substrate

Assume that some particle of elastic isotropic substrate SiO_2 participates in two kinds of the wave motion simultaneously. Let the first be caused by the pure Rayleigh wave, elliptically polarized in the sagittal (X,Z) plane ($U_2^{(R)} = 0$) with the axial ratio $|U_3^{(R)}/U_1^{(R)}| \approx 1.39$, while the second motion is caused by the shear horizontal (SH) bulk wave mode with amplitude $U_2^{(SH)} = U_B \cdot \exp(i\Phi_B)$.

Fig. 1 illustrate the corresponding changes of the rotation angles (α &β, in degrees) needed to find the resulting polarization ellipse, as well as its axial ratio, as functions of the amplitude ratio $A = U_B/\sqrt{|U_1|^2 + |U_3|^2}$ of both modes under two meanings of the phase shift $\Delta\Phi = \Phi_B - \Phi_3$ between the SH mode and the normal component of the Rayleigh wave mode $U_3 = |U_3| \cdot \exp(i\Phi_3)$.

The spatial orientation of the resulting polarization ellipse and its axial ratio can serve as a sensitive indication of the additional (caused by the SH mode) transverse motion. In this case, if $|A| < 0.2$ then the corresponding turning angles (in degrees) of the Cartesian basis are well estimated by the linear dependencies: $\beta(A) \approx 70.2 \cdot A$ for$\Delta\Phi = 0$, and $\alpha(A) \approx 97.2 \cdot A$ for$|\Delta\Phi| = \pi/2$.

It is clear that under $|A| \gg 1$ the resultant polarization tends to be the polarization of SH bulk wave mode in different ways. In particular: (a) if $\Delta\Phi = 0$, then the polarization plane turns over an X-axis ($\alpha = 0$) while $|\beta| \rightarrow 90^\circ$ & $|U_1/U_3| \rightarrow 0$; (b) if $|\Delta\Phi| = \pi/2$, then the polarization plane turns over a Z-axis ($\beta = 0$) while $|\alpha| \rightarrow 90^\circ$ & $|U_3/U_1| \rightarrow 0$.

Naturally, due to dissimilar velocities of various acoustic modes, the phase difference $\Delta\Phi$ must vary arbitrarily during their propagation along X-axis and the spatial variation of the polarization ellipse, characterizing the resultant elastic motion, should appear obligatorily. In the case of an anisotropic substrate this effect can only be analyzed with the help of precise numerical modeling tools.

4. Finite element analysis of the multimode acoustic fields in piezoelectric crystals

In any real crystal, the required components of the corresponding elastic displacements may be evaluated at any spatial point by means of the finite element analysis using the "COMSOL