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Long thickness-extensional waves in thin film bulk acoustic wave filters affected by interdigital electrodes

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ABSTRACT

We studied free vibrations of thin-film bulk acoustic wave filters with interdigital electrodes theoretically using the scalar differential equations by Tiersten and Stevens. The filters are made from AlN or ZnO films on Si substrates with ground and driving electrodes. They operate with thickness-extensional modes. The basic vibration characteristics including resonant frequencies and mode shapes were obtained. Their dependence on various geometric parameters was examined. It was found that for properly design filters there exist trapped modes whose vibrations are strong in regions with a driving electrode and decay away from the electrode edges. These trapped modes are essentially long plate thickness-extensional modes modulated by the electrode fingers. The number of trapped modes is sensitive to the geometric parameters.

1. Introduction

Thin piezoelectric films of AlN or ZnO with proper electrodes can be deposited on a silicon layer to form thin film bulk acoustic wave resonators (FBARs or TFBARs) operating in the high frequency range of GHz [1–6]. The c-axis of the material can be along the normal or the in-plane direction of the film, tilted at some angle, or even zigzag in a multilayered film [7–12]. The films can be lifted or solidly mounted on a substrate [13,14]. Structurally, FBARs are composite plate resonators of piezoelectric, metal and dielectric layers. When there are multiple driving electrodes, these plates can operate as two-port filters [15,16] or filters with interdigital electrodes [17–19].

Typical and basic theoretical models of FBARs are one-dimensional, with one spatial variable along the normal direction of the layers only. While being able to describe the most basic behaviors of FBARs through the prediction of pure thickness resonant frequencies and modes which can only exist in unbounded plates, one-dimensional models cannot describe in-plane mode variations in finite devices for which pure thickness modes do not exist. In-plane mode variations are also inherent in real FBARs with driving electrodes covering part of the piezoelectric films only. Partial electrodes are responsible for an important phenomenon call energy trapping in which the vibration is mainly confined under the partial electrodes and decays rapidly outside the electrode edges. Energy trapping is crucial to device mounting which can be designed at a distance sufficiently far away from the electrode edges so that the vibration of the device is unaffected by mounting. Energy trapping is also fundamentally important for the interactions between input and output electrodes in monolithic thin film filters with acoustic interactions. In spite of the strong need for the study of in-plane mode variations in FBARs, reported theoretical results are few and scattered, e.g., [20,21] because of the structural complexity of FBARs and the related mathematical challenges in theoretical modeling.

Tiersten and Stevens [15] derived a two-dimensional scalar differential equation that can describe the in-plane variation of the thickness-extensional operating mode and the related energy trapping in FBARs made from thin piezoelectric films on a Si layer. The scalar equation is simple and accurate. The dispersion relations of long thickness-extensional waves predicted by the scalar equation agree very well with those obtained from the three-dimensional equations of linear piezoelectricity at long wavelengths near the cutoff frequencies of the these modes, which is the frequency and wave number range of interest for the operation of FBARs. However, the equation has not been used very often. It was probably because that the derivation of the equation was very involved and one needs to go through a significant amount of algebra to calculate the coefficients of the equation in order to use it. So far the

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scalar equation has been used in the analysis of a rectangular trapped energy resonator in [15,22], a two-port filter in [15,16], and a rectangular resonator with ring electrodes for sensor application [23].

In this paper we present the results from our recent theoretical analysis using the scalar differential equation in [15] on thin film filters with interdigital electrodes for which numerical and experimental results were reported in [17–19]. We performed a free vibration analysis to obtain the most basic vibration characteristics of the filters which were not reported in [17–19]. These include the resonant frequencies, mode shapes and energy trapping.

2. Scalar differential equations for FBARs

The two-dimensional scalar differential equation governing the thickness-extensional motion of a thin AlN (or ZnO) film on a Si layer is slightly different depending on whether there is a top electrode or not. The top electrode is for electrically driving the device. In the case of a filter, the top electrodes are for electrical input and output. We consider time-harmonic motions and use the usual complex notation. All fields have the same time dependence with a common factor $\exp(i\omega t)$ which will be dropped below.

Let the normal of the filter be along the x_3 axis which is the c-axis of AlN or ZnO. When there is not a top electrode, the nth-order thickness-extensional displacement is approximately represented by [15]

$$u_3^n \cong f^n(x_1, x_2) g^n(x_3), \tag{1}$$

in which the in-plane field variation that we are interested in is described by $f^n(x_1, x_2)$ which is governed by Eq. (6.2) of [15]:

$$M_n\left(\frac{\partial^2 f^n}{\partial x_1^2} + \frac{\partial^2 f^n}{\partial x_2^2}\right) - \bar{c}_{33}^f \hat{\eta}_{f^n}^2 f^n + \rho^f \omega^2 f^n = 0. \tag{2}$$

Similarly, when there is a top electrode with a potential $V\exp(i\omega t)$ on the electrode, the corresponding equation is Eq. (6.1) of [15]:

$$M_n\left(\frac{\partial^2 f^n}{\partial x_1^2} + \frac{\partial^2 f^n}{\partial x_2^2}\right) - \bar{c}_{33}^f \bar{\eta}_{f^n}^2 f^n + \rho^f \omega^2 f^n = \rho^f \omega^2 \frac{e_{33}^f}{c_{33}^f} \frac{G_1^n}{h^f G_2^n} V. \tag{3}$$

Although (2) and (3) are simple, their coefficients need to be calculated through a series of expressions and equations scattered in a very lengthy derivation in [15]. To make it convenient, these expressions and equations needed for calculating the coefficients in (2) and (3) were summarized in the appendix of [22].

The M_n in (2) and (3) may be positive or negative. As to be seen later, the sign of M_n affects the behavior of the solutions of (2) and (3) in a fundamental way and a positive M_n is associated with the desired modes with energy trapping. In Fig. 1(a), corresponding to the ZnO FBAR in [15,22] with $n = 1$, we plot M_n versus the thickness of the piezoelectric film. The material constants of ZnO and Si were from [24]. The figure shows that M_n may change its sign, and when it does so it goes through infinity rather than zero. For the specific thicknesses of the ZnO film in [15,22] with $h^f = 15$ μm, the corresponding M_n is positive as indicated in the figure by an arrow. Similarly, for the AlN thin film filter in [17,18] with $n = 4$, M_n versus the thickness of the AlN film is shown in Fig. 1(b). The material constants of AlN were from [25]. The specific M_n of the filter in [17,18] with $h^f = 1$ μm is positive as indicated. Fig. 1 is useful in design for properly choosing the thickness of the piezoelectric film and the thickness of the Si layer for a positive M_n. Our numerical studies also showed that M_n may change its sign again for values of h^f outside the ranges shown in Fig. 1.

3. Theoretical model for thin film filters with interdigital electrodes

Consider the thin film filter with interdigital electrodes in Fig. 2. It has P parts where P is an odd number. Let p range from 1 to P. $p = 1$ represents the unelectroded part at the left edge of the plate. $p = 2,3,4,\ldots, P$-1 are the interior periodic parts either with interdigital electrodes or unelectroded, and $p = P$ is the unelectroded part at the right edge of the plate. The pth part occupies an interval $(x_1^{(p)}, x_1^{(p+1)})$ on the x_1 axis.

For filters with interdigital electrodes, the electrodes are long in the x_2 direction and the variation of the fields along x_2 is small. Therefore we neglect the small x_2 dependence. In a region without an input or output electrode, from (2), the governing equation is

$$M_n \frac{\partial^2 f^n}{\partial x_1^2} + \left(\rho^f \omega^2 - \bar{c}_{33}^f \hat{\eta}_{f^n}^2\right) f^n = 0. \tag{4}$$

If a region is with an input electrode, from (3), the governing equation is

$$M_n \frac{\partial^2 f^n}{\partial x_1^2} + \left(\rho^f \omega^2 - \bar{c}_{33}^f \bar{\eta}_{f^n}^2\right) f^n = \rho^f \omega^2 \frac{e_{33}^f}{c_{33}^f} \frac{G_1^n}{h^f G_2^n} V_{in}. \tag{5}$$

Similarly, in a region with an output electrode, from (3), the governing equation is

$$M_n \frac{\partial^2 f^n}{\partial x_1^2} + \left(\rho^f \omega^2 - \bar{c}_{33}^f \bar{\eta}_{f^n}^2\right) f^n = \rho^f \omega^2 \frac{e_{33}^f}{c_{33}^f} \frac{G_1^n}{h^f G_2^n} V_{out}. \tag{6}$$

At the left and right edges, the boundary conditions are

$$\begin{aligned} f^n &= 0, \quad x_1 = x_1^{(1)}, \\ f^n &= 0, \quad x_1 = x_1^{(P+1)}, \end{aligned} \tag{7}$$

which physically represent free edges (see the discussion following Eq. (4.50) of [15] regarding the relationship between u_3 and the relatively large stress component T_{11}). Along a line parallel to x_2 separating a two-dimensional region with a top electrode and a region without a top electrode, the continuity of f^n and its normal derivative perpendicular to the line must be continuous [15], i.e.,

$$f^n\left(x_1^{(p)-}\right) = f^n\left(x_1^{(p)+}\right), \quad \left.\frac{\partial f^n}{\partial x_1}\right|_{x_1^{(p)-}} = \left.\frac{\partial f^n}{\partial x_1}\right|_{x_1^{(p)+}}, \quad p = 2,3,4,\ldots,P, \tag{8}$$

according to Eq. (5.22) of [15].

4. Analytical solution

We consider the case of $M_n > 0$ only which has energy trapping and is useful in devices. In this case, for the pth part of the filter, if it is unelectroded, we write its solution from (4) as

$$f^n = A^{(p)} \exp(\alpha x_1) + B^{(p)} \exp(-\alpha x_1), \tag{9}$$

where $A^{(p)}$ and $B^{(p)}$ are undetermined constants and we have denoted

$$\alpha^2 = \frac{\bar{c}_{33}^f \hat{\eta}_{f^n}^2 - \rho^f \omega^2}{M_n} > 0, \tag{10}$$

which imposes an upper bound on the frequency ω. A positive M_n is needed for (10) to hold in the frequency range of interest and hence the exponential solution in (9) which is responsible for the decay of fields when there is not a top electrode (energy trapping). If the pth part is with an input electrode, its solution from (5) is

$$f^n = A^{(p)} \cos(\beta x_1) + B^{(p)} \sin(\beta x_1) + \gamma V_{in}, \tag{11}$$

where