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# Prevalence of chaos in planetary systems formed through embryo accretion

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ABSTRACT

The formation of the solar system's terrestrial planets has been numerically modeled in various works, and many other studies have been devoted to characterizing our modern planets' chaotic dynamical state. However, it is still not known whether our planets fragile chaotic state is an expected outcome of terrestrial planet accretion. We use a suite of numerical simulations to present a detailed analysis and characterization of the dynamical chaos in 145 different systems produced via terrestrial planet formation in Kaib and Cowan (2015). These systems were created in the presence of a fully formed Jupiter and Saturn, using a variety of different initial conditions. They are not meant to provide a detailed replication of the actual present solar system, but rather serve as a sample of similar systems for comparison and analysis. We find that dynamical chaos is prevalent in roughly half of the systems we form. We show that this chaos disappears in the majority of such systems when Jupiter is removed, implying that the largest source of chaos is perturbations from Jupiter. Chaos is most prevalent in systems that form 4 or 5 terrestrial planets. Additionally, an eccentric Jupiter and Saturn is shown to enhance the prevalence of chaos in systems. Furthermore, systems in our sample with a center of mass highly concentrated between ∼0.8–1.2 AU generally prove to be less chaotic than systems with more exotic mass distributions. Through the process of evolving systems to the current epoch, we show that late instabilities are quite common in our systems. Of greatest interest, many of the sources of chaos observed in our own solar system (such as the secularly driven chaos between Mercury and Jupiter) are shown to be common outcomes of terrestrial planetary formation. Thus, consistent with previous studies such as Laskar (1996), the solar system's marginally stable, chaotic state may naturally arise from the process of terrestrial planet formation.

## 1. Introduction

Our four terrestrial planets are in a curious state where they are evolving chaotically, and are only marginally stable over time (Laskar, 1996, 2008; Laskar and Gastineau, 2009). This chaos is largely driven by interactions with the 4 giant planets. However our understanding of the dynamical evolution of the gas giants, particularly Jupiter and Saturn, has changed drastically since the introduction of the Nice Model (Gomes et al., 2005; Morbidelli et al., 2005; Tsiganis et al., 2005).

The classical model of terrestrial planetary formation, where planets form from a large number of small embryos and planetesimals that interact and slowly accrete, is the basis for numerous studies of planetary evolution (e.g. Chambers, 2001a; O'Brien et al., 2006; Chambers, 2007; Raymond et al., 2009b; Kaib and Cowan, 2015). Using direct observations of proto-stellar disks (Currie et al., 2009), it is clear that free gas disappears long before the epoch when Earth's isotope record indicates the conclusion of terrestrial planetary formation (Halliday, 2008). For these reasons, a common initial condition taken when numerically forming the inner planets is a fully formed system of gas giants at their current orbital locations. Many numerical models have produced planets using this method. However, none to date have analyzed the chaotic nature of fully evolved accreted terrestrial planets up to the solar system's current epoch. It should be noted that other works have modeled the outcome of terrestrial planetary formation up to 4.5 Gyr. Laskar (2000) evolved 5000 such systems from 10000 planetesimals and showed correlations between the resulting power-law orbital spacing and the initial mass distribution. Furthermore, many works have performed integrations of the current solar system, finding solutions that showed both chaos and a very real possibility of future instabilities (Laskar, 2008; Laskar and Gastineau, 2009). Our work is unique in that we take systems formed via direct numerical integration of planetary accretion, evolve them to the solar system's age, probe for chaos and its source, and draw parallels to the actual solar system.

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Although the classical terrestrial planet formation model has succeeded in replicating many of the inner solar systems features, the mass of Mars remains largely unexplained (Chambers, 2001a; O'Brien et al., 2006; Chambers, 2007; Raymond et al., 2009b; Kaib and Cowan, 2015). Known as the Mars mass deficit problem, most simulations routinely produce Mars analogues which are too massive by about an order of magnitude. Walsh et al. (2011a) argue for an early inward, and subsequent outward migration of a fully formed Jupiter, which results in a truncation of the proto-planetary disc at 1 AU prior to terrestrial planetary formation. If correct, this "Grand Tack Model" would explain the peculiar mass distribution observed in our inner solar system. Another interesting solution involves local depletion of the disc in the vicinity of Mars's orbit (Izidoro et al., 2014). A detailed investigation of the Mars mass deficit problem is beyond the scope of this paper. It is important, however, to note that accurately reproducing the mass ratios of the terrestrial planets is a significant constraint for any successful numerical model of planetary formation.

Through dynamical modeling, we know chaos is prevalent in our solar system (Sussman and Wisdom, 1988; Laskar, 1989; Sussman and Wisdom, 1992; Laskar, 2008). It is important to note the difference between "stability" and "chaos". While a system without "chaos" can generally be considered stable, a system with "chaos" is not necessarily unstable (Milani and Nobili, 1992; Deck et al., 2013). As is convention in other works, in this paper "chaos" implies both a strong sensitivity of outcomes to specific initial conditions, and a high degree of mixing across all energetically accessible points in phase space (Deck et al., 2013) Conversely "Instability" is used to describe systems which experience specific dynamical effects such as ejections, collisions or excited eccentricities.

The chaos in our solar system mostly affects the terrestrial planets, particularly Mercury, and can cause the system to destabilize over long periods of time. Laskar (2008) even shows a 1–2% probability of Mercury's eccentricity being excited to a degree which would risk planetary collision in the next 5 Gyr. What we still don't fully understand is whether these chaotic symptoms (highly excited eccentricities, close encounters and ejection) are an expected outcome of the planetary formation process as we presently understand it, or merely a quality of our particular solar system. The work of Laskar (2000) showed us that the outcomes of semi-analytic planetary formation models of our own solar system show symptoms of chaos, and are connected to the particular initial mass distribution which is chosen. However these systems were formed without the presence of the gas giants, and planetesimal interactions were simplified to minimize computing time. Perhaps our solar system is a rare outlier in the universe, with it's nearly stable, yet inherently chaotic system of orbits occurring by pure chance. Of even greater interest, if it turns out that systems like our own are unlikely results of planetary formation, we may need to consider other mechanisms that can drive the terrestrial planets into their modern chaotic state.

This work takes 145 systems of terrestrial planets formed in Kaib and Cowan (2015) as a starting point. The systems are broken into three ensembles. The first set of 50 simulations, "Circular Jupiter and Saturn" (cjs), are formed with Jupiter and Saturn on nearly circular ($e<0.01$) orbits, at their current semi-major axes. The simulations use 100 self-interacting embryos on nearly circular and coplanar orbits between 0.5 and 4.0 AU, and 1000 smaller non-self-interacting planetesimals. The smaller planetesimals interact with the larger bodies, but not with each other. Additionally, the initial embryo spacing is uniform and embryo mass decreases with semi-major axis to yield an $r^{-3/2}$ surface density profile. The second ensemble (containing 46 integrations), "Extra Eccentric Jupiter and Saturn" (eejs) evolve from the same initial embryo configuration as cjs, with Jupiter and Saturn initially on higher ($e = 0.1$) eccentricity orbits. The final batch of integrations (49 systems), "Annulus" (ann), begin with Jupiter and Saturn in the same configuration as cjs, however no planetesimals are used. 400 Planetary embryos for ann are confined to a thin annulus between 0.7–1.0 AU, roughly representative of the conditions described following Jupiter's outward migration in the Grand Tack Model (Walsh et al., 2011b).

After advancing each system to $t = 4.5$ Gyr, we perform detailed 100 Myr simulations and probe multiple chaos indicators. By careful analysis we aim to show whether chaotic systems naturally emerge from accretion models, and whether the source of the chaos is the same as has been shown for our own solar system.

## 2. Methods

### 2.1. System formation and evolution

We use the simulations modeling terrestrial planet formation in Kaib and Cowan (2015) as a starting point for our current numerical work. In Kaib and Cowan (2015), all simulations are stopped after 200 Myrs of evolution, an integration time similar to previous studies of terrestrial planet formation (e.g. Chambers, 2001a; Raymond et al., 2004; O'Brien et al., 2006; Walsh et al., 2011b). Because we ultimately want to compare the dynamical state of our solar system (a 4.5 Gyr old planetary system) with the dynamical states of our simulated systems, we begin by integrating the systems from Kaib and Cowan (2015) from $t = 200$ Myr to $t = 4.5$ Gyr. Since bodies can evolve onto crossing orbits and collide before $t = 4.5$ Gyr, accurately handling close encounters between massive objects is essential. Thus, we use the MERCURY hybrid integrator (Chambers, 1999) to integrate our systems up to $t = 4.5$ Gyr. During these integrations, we use a 6-day timestep and remove bodies if their heliocentric distance exceeds 100 AU. Because we are unable to accurately integrate through very low pericenter passages, objects are also merged with the central star if their heliocentric distance falls below 0.1 AU. Though by no means ideal, the process of removing objects at 0.1 AU is commonplace in direct numerical models of planetary formation due to the limitations of the integrators used for such modeling. Chambers (2001a) showed that this does not affect the ability to accurately form planets in the vicinity of the actual inner solar system, since objects crossing 0.1 AU must have very high eccentricities. These excited objects interact weakly when encountering forming embryos due to their high relative velocity, and rarely contribute to embryo accretion. It should be noted that many discovered exoplanetary systems have planets with semi-major axis interior to 0.1 AU. However, we are not interested in studying such systems since we aim to draw parallels to our actual solar system. The WHFAST integrator used in the second phase of this work (Section 2.2), however, can integrate the innermost planet to arbitrarily high eccentricities, so the 0.1 AU filter is no longer used. Finally, to assess the dynamical chaos among planetary-mass bodies, any "planetesimal" particles (low-mass particles that do not gravitationally interact with each other) that still survive after 4.5 Gyrs are manually removed from the final system.

### 2.2. Numerical analysis

Numerical simulations for detailed analysis of the fully evolved systems are performed using the WHFAST integrator in the Python module Rebound (Rein and Liu, 2012). WHFAST (Rein and Tamayo, 2015) is a freely available, next generation Wisdom Holman symplectic integrator (Wisdom, 1981; Wisdom and Holman, 1991; Kinoshita et al., 1991) ideal for this project due to its reduction on the CPU hours required to accurately simulate systems of planets over long timescales. WHFAST's reduction in error arising from Jacobi coordinate transformations, incorporation of the MEGNO