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Rings of non-spherical, axisymmetric bodies

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ABSTRACT

We investigate the dynamical behavior of rings around bodies whose shapes depart considerably from that of a sphere. To this end, we have developed a new self-gravitating discrete element N-body code, and employed a local simulation method to simulate a patch of the ring. The central body is modeled as a symmetric (oblate or prolate) ellipsoid, or defined through the characteristic frequencies (circular, vertical, epicyclic) that represent its gravitational field. Through our simulations we explore how a ring's behavior – characterized by dynamical properties like impact frequency, granular temperature, number density, vertical thickness and radial width – varies with the changing gravitational potential of the central body. We also contrast properties of rings about large central bodies (e.g. Saturn) with those of smaller ones (e.g. Chariklo). Finally, we investigate how the characteristic frequencies of a central body, restricted to being a solid of revolution with an equatorial plane of symmetry, affect the ring dynamics. The latter process may be employed to qualitatively understand the dynamics of rings about any symmetric solid of revolution.

1. Introduction

Rings surrounding the gas and ice giants are one of the more fascinating sights in our solar system. These entities display subtle dynamics that lead to intricate structures, not all of which are explained. Rings also act as unique mini-laboratories, involving smaller time-scales, that could help in advancing our understanding of astrophysical disks.

Recently, Braga-Ribas et al. (2014) discovered two narrow rings about a non-spherical small-body Chariklo (equatorial radius $\sim$144.9 km and oblateness $\sim$0.213), a centaur orbiting the Sun. There is also speculation about a ring system of another minor-planet, Chiron (Ortiz et al., 2015), and it is suspected that there might have been rings around Saturn's moon Rhea, which is triaxial in shape (Jones et al., 2008; Tiscareno et al., 2010), or around Iapetus, which is oblate (Ip, 2006). Moreover, hot 'Jupiter' exo-planets are likely candidates as ring hosts and those exo-planets which are tidally locked with their parent stars may have non-axisymmetric tidal bulges. Thus, the occurrence of rings around non-spherical, irregular bodies may be a more general phenomenon than anticipated. This raises the question as to how such rings may come into existence, and whether such structures are dynamically stable or, are they transient in nature. The study of rings may also help in understanding the primary body's interior (Hedman and Nicholson, 2013).

There is much work on planetary rings, but not as much in the context of rings around non-spherical bodies like asteroids, satellites or exo-planets. Some, recent studies have investigated formation scenarios for rings around smaller bodies like Chariklo; see Pan and Wu (2016), Hyodo et al. (2016) and Araujo et al. (2016). These studies hypothesize that rings around Chariklo, or similar bodies, could come into being through tidal disruption of the central body by a planet, disruption of a previous satellite, out-gassing of material from the central body or even rotational disruption of the central body. Earlier, Dobrovolskis et al. (1989) analyzed the stability of rings in the host body's equatorial/symmetry plane, with the latter being an oblate or prolate object. They observed that, while this equatorial plane is an energy minimum for oblate bodies, it is an energy maximum for prolate bodies. However, according to them, if a debris disk does come into existence around a prolate body, it could successfully form into a ring owing to dissipative collisions, in the same manner as for a debris disk around an oblate body. In a related recent work, Lehébel and Tiscareno (2015) study the motion of a ring particle around a body that has a non-axisymmetric bulge in the equatorial plane. They show that, under some conditions on the rotation period of the central

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body, the orbital period of the particle, and the particle's orbital precessional period, the central body's bulge can be averaged and incorporated as an effective J_2 correction. Further, Gupta et al. (2016) and Michikoshi and Kokubo (2017) are one of the first *N*-body simulation studies presented in this context. This work expands upon the former.

One way to investigate the dynamics of rings is to model them as particulate systems and simulate them. Earlier works on planetary rings were concerned with simulating the complete ring system, e.g. works by Trulsen (1971, 1972a, 1972b), Brahic (1975, 1977) and Hameen-Anttila (1978, 1981, 1982). These early simulations evolved the entire trajectory of all constituent ring particles; these particles were large in size and few in number. Though these simulations gave preliminary insights into the ring dynamics, they did not resolve finer dynamical details. Even today, the number of particles considered ranges between $10^3 - 10^6$, depending on model complexity and the corresponding computational costs. However, even the higher limit of this range is tiny in comparison to the particle distribution in a typical ring system. Wisdom and Tremaine (1988) proposed a local cell simulation method in planetary ring simulations, which allowed the consideration of a much larger number of particles. Such simulations have allowed further insights into the ring dynamics. Recent studies, like Salo (1995), Salo and Schmidt (2010), Lewis (2001), Lewis and Stewart (2000), Perrine et al. (2011) etc., have primarily followed this simulation methodology.

In this work, we numerically investigate the dynamics around non-spherical bodies, specifically symmetric ellipsoids that may be oblate or prolate. We also employ the local simulation method wherein the particles are restricted to a small patch and periodic boundary conditions are applied in the plane of the ring. In addition to the gravity of the central body, the particles in the patch are allowed to interact through self-gravity and collisions. To account for collisions, we modified the soft-sphere discrete element method based code of Bhateja et al. (2016). This code uses the spring-dashpot model proposed by Cundall and Strack (1979) to model inelastic collisions between particles. The particles under this study are assumed to be smooth.

The paper is divided into four parts. In the next section, we discuss our simulation methodology. In Section 3 we briefly present details on how the code is implemented and in Section 4 we present how the simulations are characterized. Following this, we discuss the results of our study in Sections 5 and 6. We conclude in Section 7 with a brief summary and thoughts about future work. Appendices present additional details of our simulation algorithm, code validation and the relevant parameters of study.

2. Simulation methodology

2.1. Test-Section

We follow the local simulation method and focus on a 'Test-Section' (TS) with N particles that is orbiting about a central/primary body; see Fig. 1. This central body is assumed to have a ring about it and, at any instant of time, the particles contained in the TS represent those in a small patch of the ring. The central body is such that it produces an axisymmetric gravitational field that is also symmetric about the body's equatorial plane. The equatorial plane is also the mid-plane of the ring.

The TS is a rectangular box whose mid-plane coincides with the central body's equatorial plane. The spatial position of the TS is defined by its centroid C, also referred to as the 'guiding center'. This point C is situated at a fixed radial distance r_{gc} from the center of the central body and moves on a circular orbit at the corresponding Keplerian angular velocity Ω; this defines the orbiting velocity of the TS.

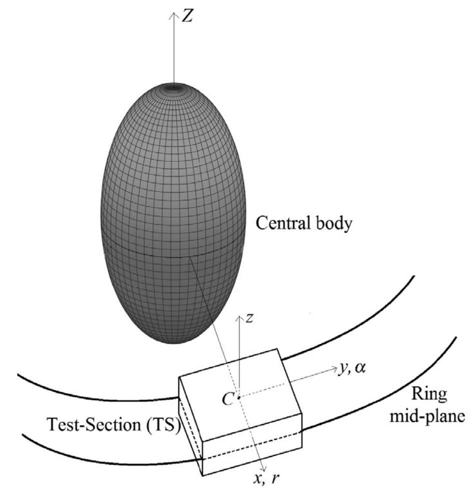

Fig. 1. Schematic representation of the test-section (TS) orbiting about an ellipsoidal central body. Also shown is a cylindrical coordinate system (r, α, Z) with origin O at the center of the central body, and a Cartesian coordinate system (x, y, z) with origin C (guiding center) at the center of the TS and which orbits with the TS. The axes Z and z are parallel.

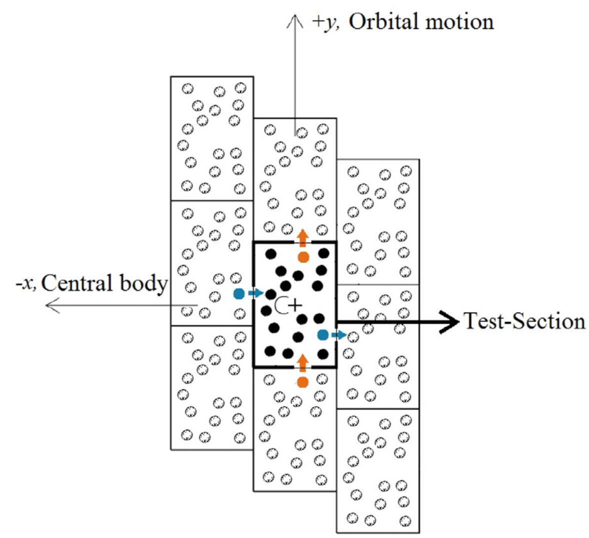

Fig. 2. Schematic representation of the test-section (TS) when viewed normal to plane of the ring along the z direction. The TS is surrounded by identical images because of the imposed periodic boundary conditions; cf. Section 2.3.1. Initially, boundaries of all the sections are aligned. However, they get misaligned with time due to Keplerian shear caused by the orbital motion of the TS; see Section 2.3.1 for more details. The figure also depicts particle movement across boundaries normal to the x- (blue) and y-(orange) axes, with the arrows indicating their direction of motion.

The motion of particles inside the TS is described by a rotating Cartesian coordinate system (x, y, z) centered at C; see Figs. 1 and 2. The x-axis points in the radial direction away from the central body, the y-axis is in the direction of the orbital motion of the TS, and the z-axis is normal to the equatorial plane. The TS is oriented such that, at any time, its three faces are perpendicular to the three axes. The z-direction is unbounded. The dimensions of the TS in x and y directions are x_{TS} and y_{TS}, respectively. These dimensions are much smaller than the orbital radius r_{gc}. This justifies our ignoring the curvature of ring and, consequently, linearizing the equations governing particle motion in the TS. The dimensions of the TS must, however, be large enough to not allow any significant correlation in the dynamics of particles present at its opposite faces.