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Time-dependent rotational stability of dynamic planets with viscoelastic lithospheres

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ABSTRACT

We extend previous work to derive a non-linear rotational stability theory governing true polar wander (TPW) of terrestrial planets with viscoelastic lithospheres. We demonstrate, analytically and using numerical examples, that our expressions are consistent with previous results in the limiting cases of low and infinite (i.e., purely elastic) viscosity lithospheres. To illustrate the stability theory, we compute TPW on Mars driven by a simple, prescribed mass loading. Our calculations demonstrate that on short time scales relative to the relaxation time of the viscoelastic lithosphere, the rotation axis follows a constrained path that reflects stabilization by remnant strength in the lithosphere, but that on long times scales this stabilization disappears and the load ultimately reaches the equator. Earlier work based on the assumption of a permanent remnant bulge in the case of a purely elastic lithosphere has suggested that Martian TPW would not persist for any significant time after a load is emplaced, and thus an equilibrium stability theory is sufficient to model long-term (order 1 Myr or longer) polar motion of the planet. Our results suggest, in contrast, that TPW on Mars can continue over time scales on the order of the relaxation time of the lithosphere after load emplacement; for sufficiently high lithospheric viscosities, this time scale may be comparable to the age of the planet.

1. Introduction

Modern theories describing the rotational stability of terrestrial planets emerged from the insights of Gold (1955). He argued that the rotation axis of all such planets is inherently unstable because any stabilization provided by the rotational bulge is transient; that is, the rotational bulge will ultimately reorient perfectly to any new position of the rotation axis and the system will thus lose all memory of any prior rotational state. In this case, any isolated mass load will eventually migrate, through true polar wander (TPW), to the equator, and, furthermore, a set of evolving loads can trigger relatively rapid and large ($\sim 90^\circ$) amplitude TPW events (Goldreich and Toomre, 1969).

Willemann (1984) introduced an important caveat to Gold's arguments. He argued that TPW would introduce permanent elastic stresses in an otherwise unstressed planetary lithosphere, and this so-called remnant bulge would introduce a memory of prior rotational states. The net TPW driven by a mass load would, in this case, be governed by a balance between the centrifugal forces driving the mass load toward the equator and the stabilization of the rotation axis associated with the remnant bulge. Willemann's original derivation predicted that the remnant bulge stabilization was independent of the elastic thickness of the lithosphere. However, Matsuyama et al. (2006) corrected several minor errors in the derivation and his final expression for TPW revealed an important dependence on this thickness.

The above studies all considered the equilibrium position of the rotation axis; that is, the final position of the pole after all viscous stresses within the planet had fully relaxed. The speed of TPW will be a strong function of the mass of the evolving loads and the viscosity of the planetary interior, and various methods have been developed to incorporate this time history into a rotational stability theory in the absence of remnant bulge stabilization (e.g., Ricard et al., 1993; Tsai and Stevenson, 2007). Chan et al. (2011) adapted a linearized theory of TPW developed to study ice age Earth rotation (Sabadini and Peltier, 1981; Sabadini et al., 1982; Wu and Peltier, 1984; Vermeersen and Sabadini, 1999; Mitrovica et al., 2005) to consider the impact of remnant bulge stabilization on time-dependent polar wander. Their theory was valid for small displacements of the pole, and they demonstrated that the range of validity was a strong function of the location of the loads relative to the rotation axis. To overcome this small-angle limitation,

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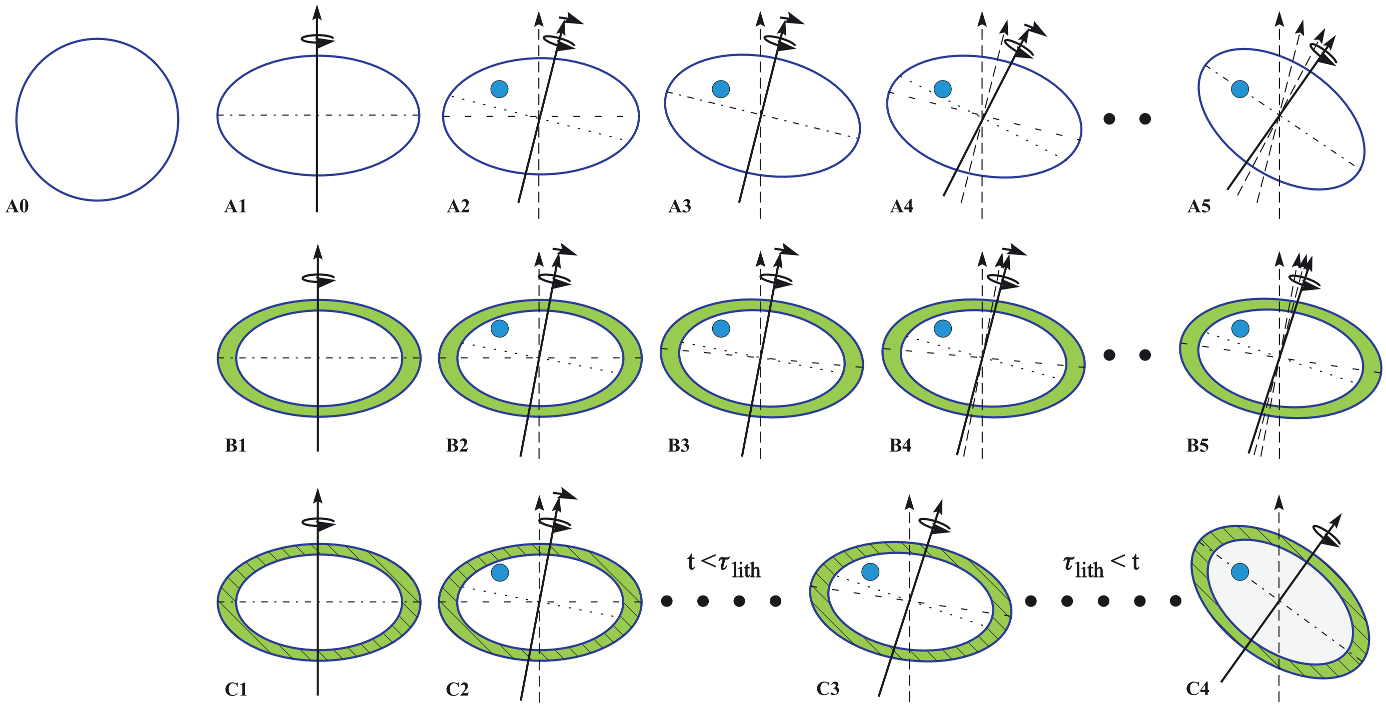

Fig. 1. Schematic illustration of the physics of long-term TPW on terrestrial planets (A) without a lithosphere, (B) with an elastic lithosphere (represented by the solid green shell), and (C) with a viscoelastic lithosphere (green shell with hatching). All frames are drawn in a reference frame fixed to the load within the interior of the planet (blue dot). The solid arrow in each frame (other than A0) represents the contemporaneous rotation axis, while the dashed arrows symbolize previous locations of the rotation axis. The dotted line in the interior of the planet denotes the equator (i.e., the great circle 90° from the rotation pole), while the dashed line is positioned to pass through the equator of the oblate form of the planet (i.e., it passes through the planet at its widest girth). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Harada (2012), Creveling et al. (2012) and Chan et al. (2014) extended the pioneering approach of Ricard et al. (1993) to incorporate the effects of the remnant bulge on the predicted TPW history. In these studies, the lithosphere was treated as a purely elastic layer, with effectively infinite viscosity.

In reality, a lithosphere will have finite viscosity, and it will viscously relax in the case of a loading of sufficiently long time scale, and no rotational stability theory for terrestrial planets has yet considered this more realistic case. In this study, we extend the TPW theories of Ricard et al. (1993), Harada (2012), Creveling et al. (2012) and Chan et al. (2014) to include stabilization by a viscoelastic lithosphere of high (but finite) viscosity. On such a planet, TPW driven by loading of short time scale (i.e., a time scale shorter than the viscous relaxation time of the lithosphere) will be strongly stabilized by a remnant bulge, while on long time scales the lithosphere will provide no such stabilization and the physics described by Gold (1955) and Goldreich and Toomre (1969) will prevail. Our generalized theory accommodates both scenarios, and, more importantly, the stabilization associated with intermediate time scale loading.

In the next section we provide a short, schematic description of load-induced TPW on terrestrial planets in the case of no lithosphere, an elastic lithosphere, and a viscoelastic lithosphere. Following this, we derive the equations governing TPW in the last of these cases and consider the limiting cases of no lithosphere and an elastic lithosphere. We end with a section showing some numerical examples of Martian TPW that illustrate the theory.

2. The physics of TPW in the presence of a viscoelastic lithosphere

Gold (1955) considered TPW induced by the loading (i.e., mass redistribution) of a terrestrial planet possessing no lithosphere, as depicted in Fig. 1(A). To begin, we consider an initially non-rotating, spherical planet (A0). Next, the planet is spun up to some rotation rate and a rotational bulge is created that ultimately achieves an equilibrium form (A1). If a positive mass load is applied (A2), the rotation axis begins to adjust by moving away from this forcing (equivalently, in a reference frame fixed to the rotation axis, the load is thrown outwards toward the equator). This polar motion is initially resisted by the rotational bulge, but on a fluid planet the bulge will relax to match the new rotational state (A3). This process continues incrementally (A4) until, in the limit of long time, the load reaches the equator (A5). The rotation axis in the model of Gold is thus inherently unstable. While this model is only concerned with the final (equilibrium) form of the rotating body, Ricard et al. (1993) developed a non-linear rotational stability theory that mathematically captures the time dependence of the system associated with the evolution from (A1)–(A5). The theory assumes that the loading and response is slow enough that the rotation axis remains aligned with the principle axis of inertia.

Next, we consider TPW in the presence of an elastic (effectively infinite viscosity) lithosphere, the case treated by Willemann (1984). We begin with Fig. 1(A1), but assume that the planet develops an unstressed elastic lithosphere through slow cooling (B1). When a load is applied (B2), the rotation axis adjusts as before (the load is thrown toward the equator), but the polar motion is now resisted by elastic stresses induced in the lithosphere by the perturbed orientation of the centrifugal potential, i.e., the net TPW from the initial state is greater in (A2) than (B2). Willemann (1984) called this resistance the ‘remnant bulge’, since it will exist at any time in which the rotational bulge (or rotation axis) departs from its initial orientation. This process of gradual adjustment of rotation axis and reorientation of the bulge will continue (B3–B5) until the elastic stresses in the lithosphere balance the load-induced forcing. In the final state (B5), the load will not reach