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On the dynamics, existence of chaos, control and synchronization of a novel complex chaotic system

K. Vishal [a,*], Saurabh K. Agrawal [b]

[a] Department of Mathematics and Basic Science, NIIT University, Neemrana, Rajasthan-301705, India
[b] Department of Applied Sciences, Bharati Vidyapeeths College of Engineering, New Delhi-110063, India

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ABSTRACT

In this paper, the authors have studied the dynamics of a novel complex chaotic system with fractional order derivative and found the existence of chaos. The novel complex system is simulated for integer as well as fractional orders which shows some unusual phenomena. The main contribution of this effort is an implementation of the Largest Lyapunov Exponent (LLE) criteria based on Wolf's algorithm. The conditions for chaos control based on the fractional Routh–Hurwitz stability conditions and feedback control are given. Also synchronization between a fractional order novel chaotic system and a controlled fractional order novel system using the modified adaptive projective synchronization method for different scaling matrices has been obtained. Numerical simulation results are carried out using the Adams–Bashforth–Moulton method.

1. Introduction

The concept of a dynamical system, which has been evolved from Newtonian mechanics, has tremendous application in science and engineering. Mathematical modeling of dynamical systems shows the evolution of a system in terms of an equation of motion. The existence of chaos is an interesting phenomenon associated with a nonlinear dynamical system, which indicates the occurrence of an irregular solution while the equation of motion is deterministic. A dynamical system is said to be chaotic if two arbitrarily close starting points diverge exponentially, with the results that the future behavior is eventually unpredictable. The behavior of dynamical systems is exponentially sensitive to the initial condition, which is popularly known as the butterfly effect. In 1963, Lorentz [1] found the first canonical chaotic attractor. Various dynamics, the existence of chaos and its synchronization have been studied on a large number of real dynamical systems. A dynamical system which involves real variables is said to be a real dynamical system. However, there are also many real life examples involving complex variables. The complex Lorenz equations have been firstly designed by Fowler et al. [2], in which they have discussed the significance of the complex Lorenz equation in relation to real fluid dynamical processes. Chaos control, synchronization and dynamics have been studied for several chaotic complex systems during the last few decades. Nowadays researchers are applying the concept of complex dynamical systems into engineering, such as laser systems, electromagnetic fields, and thermal convection of liquid flows [3–5]. Similarly the complex Chen and complex Lu systems have

* Corresponding author.
E-mail address: kumarvishal.bhu@gmail.com (K. Vishal).

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been investigated [6]. There are many research results which have been proposed about the dynamical properties of chaotic systems in real space as well as complex space [7–10].

A dynamical system involving fractional time derivatives instead of integer order time derivatives is known as a fractional dynamical system. A fractional order system which is a generalization of an integer order system describes the memory effect [11]. It has been extensively applied for the modeling of many real problems appearing in viscoelasticity, power-law phenomenon in fluid mechanics, biology, ecology, dielectric polarization, electromagnetic waves, quantum evolution of complex systems, fractional kinetics [12–15], and so on.

Motivated by the above discussion, in this article, a novel fractional-order complex system is proposed. Recently, Sun et al. [16] have discussed the dynamical properties and combination-combination complex synchronization for the said system with integer order derivative. Dynamical properties and existence of chaos are investigated with varying the fractional derivative orders with the help of the Largest Lyapunov Exponents criteria based on Wolf's algorithm [17,18]. Recently the same technique has been applied for a fractional order macroeconomic model [19]. We know that chaotic systems are much too sensitive to the initial conditions, because of this synchronization of such systems is very difficult. In recent years, several applications of chaos synchronization have been proposed and found in the literature. Ott et al. [20] gave the famous OGY control method in 1990 to control the chaotic behavior, and in the same year a pioneer work on synchronization by Pecora and Carroll was done [21]. After the work of Pecora and Carroll many types of synchronization phenomena have been discovered, such as complete synchronization [22], phase synchronization [23], projective synchronization [24], function projective synchronization [25] and so on. Recently, Agrawal and Das [26] introduced a modification on the adaptive projective synchronization with unknown parameters for fractional order chaotic systems. Recently Luo and Zeng have discussed the control and synchronization of chaotic systems via states recovery and with novel input [27–29]. In the present article a sincere attempt is taken to find the existence of chaos in the novel fractional order complex dynamical systems with the help of the LLE. Computer simulation is carried out using the Adams–Bashforth–Moulton method [30,31].

This article is organized in the following manner. In Section 2, some basics of the fractional calculus and stability conditions of fractional order systems are given. In Section 3, a fractional-order complex system is proposed, the dynamical behavior of the system is studied and the existence of chaos is found with the help of the LLE. In Section 4, the conditions to control the chaos have been provided. In Section 5, a synchronization scheme for the novel system is proposed. In Section 6, the simulation results for the synchronization are given. Finally a concluding remark is given in Section 7.

2. Some preliminaries and stability conditions

In this section some basic definitions of fractional order derivatives are given. Also stability conditions for fraction order system are given in terms of Routh–Hurwitz determinants.

2.1. Fractional calculus

Definition 1. A real function $f(t)$, $t > 0$, is said to be in the space C_μ, $\mu \in R$, if there exists a real number $p > \mu$, such that $f(t) = t^p f_1(t)$, where $f_1(t) \in C(0, \infty)$, and it is said to be in the space C_μ^n if and only if $f^{(n)} \in C_\mu$, $n \in N$.

Definition 2. The Riemann–Liouville fractional integral operator (J_t^q) of order $q \geq 0$, of a function $f \in C_\mu, \mu \geq -1$, is defined as

$$J_t^q f(t) = \frac{1}{\Gamma(q)} \int_0^t (t-\xi)^{q-1} f(\xi) d\xi, q > 0, t > 0, \tag{1}$$

where $\Gamma(q)$ is the well-known gamma function.

Definition 3. The fractional derivative D_t^q of $f(t)$, in the Caputo sense is defined as

$$D_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t (t-\xi)^{n-q-1} f^n(\xi) d\xi, \tag{2}$$

for $n-1 < q < n$, $n \in N$, $t > 0$, $f \in C_{-1}^n$.

The important reason for choosing the Caputo derivatives for solving initial value fractional order differential equations is that the Caputo derivative of a constant is zero, whereas the Riemann– Liouville fractional derivative of a constant is not equal to zero.

2.2. Stability of the system

Consider a five-dimensional fractional order system

$$D_t^q x_1(t) = f_1(x_1, x_2, x_3, x_4, x_5),$$
$$D_t^q x_2(t) = f_2(x_1, x_2, x_3, x_4, x_5),$$
$$D_t^q x_3(t) = f_3(x_1, x_2, x_3, x_4, x_5),$$

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