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Scaling law for voltage–current curve of a superconductor tape with a power-law dependence of electric field on a magnetic-field-dependent sheet current density

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ABSTRACT

Systematic theoretical study on the voltage (V) vs. current (I) curves of high-T_c superconductor (HTS) thin tapes has not been done till now, although their measurements are frequently used for determining critical current I_c at electric field $E \equiv V/l_v = E_c = 10^{-4}$ V/m, l_v being the voltage tap distance. On the other hand, it is well recognized that such tapes obey a power-law dependence of local electric field on local sheet current density with a Kim-model critical sheet current density, from which the V vs. I curve may be calculated as a function of current ramp rate R. Such calculations are carried out in the present work with a scaling law deduced, which states that if E/E_c vs. I/I_c is a solution at given apparent power-law exponent n_a and R/E_c, then this R/E_c multiplied by a constant C leads to another solution with E/E_c and I/I_c multiplied by C and C^{1/n_a}, respectively. In the help of the scaling law, condition-dependent V vs. I may be studied systematically and completely based on a limited amount of numerical computations and $V-I$ curve measurements may be performed under properly controlled conditions to become a more powerful tool for HTS research.

1. Introduction

The most important electromagnetic quantity of a high-temperature superconductor (HTS) tape is its critical current I_c at $T = 77$ K determined from voltage ($V \equiv El_v$) -current (I) curve, l_v and E being voltage tap distance and the electric field corresponding to V, at $E = E_c$ with $E_c = 10^{-4}$ V/m being often defined. The E vs. I curve around $E = E_c$ usually shows a power-law relation $E/E_c = (I/I_c)^n$, which is the consequence of a power-law dependence of local electric field $\mathbf{E}$ on local current density $\mathbf{J}$ or, for the case of thin tape of thickness δ, on local sheet current density $\mathbf{K} \equiv \mathbf{J}\delta$,

$$\mathbf{E} = (E_c/J_c)|J/J_c|^{n-1}\mathbf{J}, \quad (1)$$

$$\mathbf{E} = (E_c/K_c)|K/K_c|^{n-1}\mathbf{K}, \quad (2)$$

where J_c and K_c are critical current density and critical sheet current density occurring at $E = E_c$.

Eq. (1) has been quite accurately confirmed for HTSs in a wide E range from 10^{-9} to 10^{-4} V/m by Mawatari et al. [1–3] and explicitly derived by Brandt from the collective flux creep theory for a special case, with n being named as flux creep exponent and indicating the height of thermal activation energy barrier for vortex depinning [4,5]. After Bean proposed the critical state (CS) model with a constant J_c (the Bean model) to explain magnetic properties of hard superconductors [6,7], Kim et al. proposed a local field dependent J_c (the Kim model) to simulate magnetization curves of a type-II superconductor ring [8], which was successively explained by Anderson by a model of vortex pinning and creep with the field dependence indicating pinning center density [9,10]. Thus, when Eq. (1) is used to express the flux creep behavior, a local field dependent J_c should be incorporated, for which a Kim-model $J_c(H)$ has been a good choice [4,11].

The above should be the commonly accepted basic knowledge on the current–voltage curves of HTS tapes up to date. Correspondingly, the results of all previous E vs. I curve measurements include the values of I_c, n, and, if a perpendicular bias magnetic field H was also applied, the $I_c(H)$ relation. All these results are obtained at E around or above E_c, and the significance of the E vs. I curve in the region of $E \ll E_c$ have been overlooked, or sometimes considered as erroneous, owing to the smallness of the measured voltage. However, many experiments show that the E vs. I curves of HTS samples are often reproducible with measured voltage at the

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lowest E above the resolution of the nanovoltmeter used. Thus, it is urgently needed to have theoretical results of E vs. I curves to be compared with experiments and to guide experimentalists for their further research in order to get additional information from better controlled measurements.

In fact, the E vs. I curve of a cylinder of radius a obeying a power-law $E(J)$ relation has been systematically studied in [12,13], where the field dependence of J_c is not considered. It is concluded that starting from a zero-field cooled state with ramping I at a positive rate R, the supercurrents penetrate from the surface to the center and the electric field at surface, $E(a)$, increases in proportion to I at small I and then gradually approaches a relation of $E \propto I^n$. Especially, a scaling law has been deduced, which states that if R/E_c is multiplied by a constant C then I/I_c and E/E_c are multiplied by $C^{1/(n-1)}$ and $C^{n/(n-1)}$, respectively. The importance of such a scaling law is clear; the theoretical study on E vs. I curves has to be done by numerical computations under many different multi-parameter conditions, and it can be done completely and systematically with a minimum amount of computations under optimally chosen conditions only when an accurate scaling law is deduced.

A similar study for a thin tape obeying Eq. (2) will be carried out in the present work with a Kim-model $K_c(H)$ included. A scaling law will be deduced, which states that if E/E_c vs. I/I_c is a solution at given n and R/E_c, then this R/E_c multiplied by a constant C leads to another solution with E/E_c and I/I_c being multiplied by C and $C^{1/n}$, respectively, if K_c in Eq. (2) is a constant, or generally by C and C^{1/n_a}, respectively, where n_a is an apparent flux creep exponent equal to or larger than n, if a Kim-model $K_c(H)$ is considered.

The Kim-model $K_c(H)$ and its corresponding critical current I_c are expressed in Section 2. The formulas with quantities in SI units used in numerical computation of E vs. I curves and the formulas with dimensionless quantities useful for deducing the scaling law are given in Section 3. The scaling law is described and demonstrated in Section 4, and it is a basis for condition-dependent study of E vs. I curves as explained in Section 5. The relevance of the present work to experiments of HTS tapes is shown in Section 6. Recommended by our referees, Section 7 is added for a further discussion on the obtained results before the conclusions stated in Section 8. To be compared with the present numerical computation with finite n, E vs. I for the CS case of $n \to \infty$ and constant K_c is derived analytically in Appendix A. The scaling law deduced in the present work is compared with those deduced earlier for hysteresis loop, ac susceptibility, ac loss, and voltage–current curve in Appendix B.

2. Kim-model $K_c(H)$ and critical current

The Kim model $J_c(H)$ was the first field dependent CS model and was used extensively in the study of conventional superconductors. Complete Kim-model derivation and calculation of magnetization and ac susceptibility of long samples of rectangular or circular cross-section were performed by Chen et al. until the discovery of HTSs [14,15]. The model was originally expressed in the form of $J_c(H) = k(H_0 + |H|)^{-1}$, where k and H_0 are positive constants [8,14]. It was improved in a systematic study on ac susceptibility as $J_c(H) = J_0(1 + |H|/H_K)^{-1}$, where H_K is a positive characteristic field [16]. With this expression, for an infinite slab of thickness $2a$, a dimensionless parameter p introduced in [14] was written $p_K = (2J_0a/H_K)^{1/2}$, so that another expression may be written as

$$J_c(H) = J_0[1 + p_K^2|H|/(2J_0a)]^{-1}. \tag{3}$$

The thin tape studied is placed along the z axis located at $|y| \le \delta/2$ and $|x| \le a \gg \delta$. With sheet current density K and referring to Eq. (2), we write the Kim model as

$$K_c(H) = K_0[1 + p_K^2|H|/(K_0/\pi)]^{-1}. \tag{4}$$

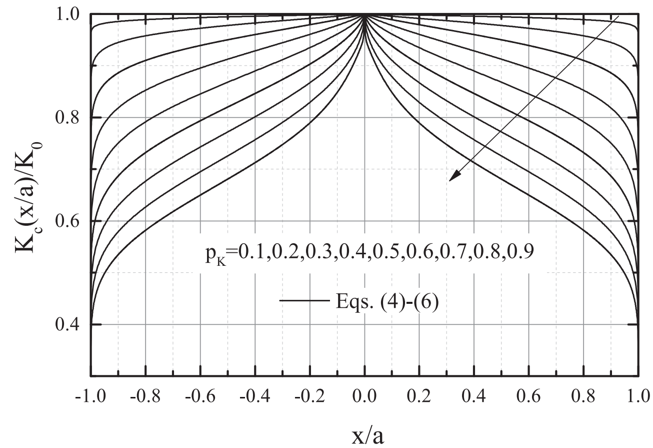

Fig. 1. Critical sheet current density profiles for $p_K = 0.1, 0.2, \ldots, 0.9$ calculated using Eqs. (5)–(7). Arrow indicates the direction of increasing p_K.

The quantity K_0/π in Eq. (4) is the counterpart of $2J_0a$ in Eq. (3). Physically, J_0a is the full penetration field H_p of a slab of thickness $2a$ obeying the Bean model with a constant $J_c = J_0$ magnetized by a longitudinal applied field H_a. Since $H_p \to \infty$ for a thin tape of width $2a$ and thickness $\delta \to 0$ obeying the Bean model with a constant $K_c = K_0$ when magnetized by a perpendicular applied field H_a, a characteristic field $H_c = K_0/\pi$ has been introduced by Brandt and Indenbom for calculating perpendicular magnetization [17].

Dividing the width $2a$ into N equal elements, each centered at $x_i = [(2i-1)/N - 1]a(i = 1, 2, \ldots, N)$, $K_c(x_i)$ is written $K_{c,\,i}$ and may be numerically calculated by

$$K_{c,i} = K_0[1 + p_K^2|H_i|/(K_0/\pi)]^{-1}, \tag{5}$$

where H_i is $H(x_i)$ and calculated by

$$H_i = H_a + \frac{2a}{N}\sum_{j=1,\,j\neq i}^{N} P_{ij}K_{c,j}, \tag{6}$$

$$P_{ij} = [2\pi(x_i - x_j)]^{-1}. \tag{7}$$

Starting from $K_{c,i} = K_0$ for $i = 1, 2, \ldots, N(= 1601)$, the K_c profiles at $H_a = 0$ for $p_K = 0.1$–0.9 are calculated iteratively with results shown in Fig. 1, from which the critical current I_c is obtained by integration of each K_c profile. The following fitting formula is found to have accuracy better than 0.1%,

$$2aK_0/I_c = [1 + (p_K/0.847)^2]^{1/2}. \tag{8}$$

The same expression for Kim-model $K_c(H)$ has been used in a study of the scaling law for ac susceptibility [18].

3. Equations used for calculation and scaling of $E(x)$ vs. I curves

3.1. Calculation of $E(x)$ vs. I curves

The E vs. I defined in Section 1 should be changed as $E(x)$ vs. I since E is also position dependent as shown by the following calculation. The calculation of $E(x)$ vs. I curves is carried out in a similar way to that in [19,20]. The formulas used in the computation program are given below.

Dividing the width $2a$ into $N(>200)$ equal rectangular elements as above, the components of the 1D vector potential produced by sheet currents in the Coulomb gauge [21], $A_{K,i}(i = 1, 2, \ldots, N)$, are expressed by

$$A_{K,i} = -\frac{2\mu_0 a}{N}\sum_{j=1}^{N} Q_{ij}K_j, \tag{9}$$