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Estimating the mass variance in neutron multiplicity counting—A comparison of approaches

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ABSTRACT

In the standard practice of neutron multiplicity counting , the first three sampled factorial moments of the event triggered neutron count distribution are used to quantify the three main neutron source terms: the spontaneous fissile material effective mass, the relative (α, n) production and the induced fission source responsible for multiplication.

This study compares three methods to quantify the statistical uncertainty of the estimated mass: the bootstrap method, propagation of variance through moments, and statistical analysis of cycle data method. Each of the three methods was implemented on a set of four different NMC measurements, held at the JRC-laboratory in Ispra, Italy, sampling four different Pu samples in a standard Plutonium Scrap Multiplicity Counter (PSMC) well counter.

1. Introduction

In the standard practice of Neutron Multiplicity Counting (NMC), the first three sampled factorial moments of the event triggered neutron count distribution are used in an inversion model to extract the spontaneous fission rate, the (α, n) rate and the multiplication of the item. A significant advantage of NMC over other nondestructive assay methods is the relative transparency of structural materials to neutrons, making it a useful method when sampling impure, poorly characterized items.

As in any experimental method, uncertainty estimation is an inherent part of the measurement, and no result is complete without it. Yet, at present, there is no comprehensive guide regarding how to estimate the uncertainty of the measured mass using NMC.

Typically, "uncertainties" can be divide into three categories: uncertainties in the physical parameters (such as detection efficiency, the prompt fission multiplicity distributions etc.), systematic errors (due to model assumptions – such as the single energy point model and neglecting the delayed neutrons – or due to numeric methods) and the statistical uncertainty due to the random nature of neutron counting (and fractionally larger when sampling the higher moments).

From an operational point of view, understanding the statistical error has high importance for two main reasons: first, sampling high moments of the count distribution is vulnerable to a large statistical uncertainty. Second, out of all the uncertainty factors mentioned , the statistical uncertainty is the only one the user can control by extending the duration of the measurement.

The objective of the present study is to perform a comparison between three methods for estimating the statistical uncertainty of the estimated mass: the bootstrap method, propagation of variance through moments and statistical analysis of cycle data.

The comparison was done experimentally. Each of the three methods was implemented on a set of four NMC measurements, held at the JRC-laboratory in Ispra, Italy, sampling four different Pu samples in a Plutonium Scrap Multiplicity Counter (PSMC) well counter [1]. In order to create a reference value, the measurement was repeated for a sufficient number of times (30–90), and the statistical spread of the repetitions was used as the reference value.

The paper is arranged in the following manner: Section 2 gives the necessary background on NMC and give an overview of the paper. Section 3 describes the different methods used to estimate the statistical uncertainty. Section 4 describes the experimental setting and introduce and explain the reference values used for comparison. Section 5 describes the experimental results, and Section 6 concludes.

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2. Neutron multiplicity counting

2.1. Neutron multiplicity counting and the SVM method

Most spontaneous fissile materials emit neutrons in a known rate (per mass unit). Thus, in a system with a known detection efficiency, the mass of the spontaneous fissile material is propositional to the average count rate of the spontaneous fission neutrons in a known proportion. However, such simple consideration only provides a partial solution, since the count rate of the neutron detections is highly influenced by two additional neutron sources: (α, n) reactions in sample impurities, and induced fissions (typically in the odd Plutonium isotopes). Moreover, since the detection system is often based on ^{3}He proportional counters imbedded in a moderating medium, variations in the energy spectrum between the different neutron sources have a negligible effect on the counter efficiency or the die away time, and the neutrons cannot be distinguished through energetic considerations.

On the other hand, since the three sources have a different statistical nature, the contribution of each source can be quantified by measuring higher moments of the count distribution. Such general considerations are referred to as Neutron Multiplicity Counting (NMC) or Time Interval Analysis (TIA).

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The shift register method is routinely used in NMC [2], where the so called Singles, Doubles and Triples rate are used to quantify the three neutron sources. Other methods include the Random Trigger Interval (RTI) method [3] and the Skewness–Variance–Mean (SVM) method [4]. Because all methods, eventually, sample the first three moments of the count distribution (although through different random variables), all methods are mathematically equivalent[1] [5].

Since the outline of the present study is estimating the statistical uncertainty in the observables – and the final mass result – our choice is the SVM method, where the sampled quantities are very simple: the first three central moments of the number of detections in consecutive (fixed) gates.

In more detail, the SVM method is implemented in the following manner: the measurement (of duration of T_{tot}) is divided into N consecutive gates of duration T (where T is typically on the order of the system neutron die away time, and $N = T_{tot}/T \gg 1$). Denoting the number of neutron detections in the k_{th} gate ($1 \leq k \leq N$) by X_k, the sample mean is given by $\widehat{E}(X) = \frac{1}{N}\sum_{k=1}^{N} X_k$, sample variance is evaluated through $\widehat{Var}(X) = \frac{1}{N-1}\sum_{k=1}^{N}\left(X_k - E(X)\right)^2$ and the skewness by $\widehat{Sk}(X) = \frac{1}{N-1}\sum_{k=1}^{N}\left(X_k - E(X)\right)^3$[2]. Once the sampling is done, the generalized factorial neutron multiplicity moments – defined as the factorial moments of the number of neutron emitted in an entire fission chain starting with a single source event – are related to the sampled moments by [4]:

$$
\begin{aligned}
D_{G,1} &= \frac{\widehat{E}(X)}{SP_dT} \\
D_{G,2} &= \frac{\left(\widehat{Var}(X) - \widehat{E}(X)\right)}{SP_d^2(e^{-\lambda T} - 1 + \lambda T)/\lambda} \\
D_{G,3} &= \frac{\left(\widehat{Sk}(X) - 3\widehat{Var}(X) + 2\widehat{E}(X)\right)}{SP_d^3(e^{-2\lambda T} + e^{-\lambda T} - 3 + 2\lambda T)/2\lambda}
\end{aligned}
\tag{2.1}
$$

where P_d is the detection efficiency (the probability that an emergent neutron will be detected), S is the source rate—the number of source events (spontaneous fissions or (α, n)) per time unit and λ is the reciprocal of the detector system die away time, adopting the exponential model.

Finally, the generalized factorial moments are used to quantify the different neutrons sources through the so called "Bohnel Method" [6,7], describing the generalized factorial moments in term of the following parameters:

1. The spontaneous fission fraction U: the fraction of the source that is due to spontaneous fissions only[3]
2. The leakage multiplication factor M_L: the neutron leakage multiplication factor, defined as the product between the total multiplication and the probability of neutron leakage [8].
3. $D_{sf,n}, D_{if,n}$ The n_{th} factorial moments of the neutron emission distribution in a spontaneous/induced fission (respectively).

Denoting by $D_{G,\ell} = D_{G,\ell}(U, M_L)$ the ℓ_{th} factorial moment of the distribution of the number of neutron emitted in an entire fission ignited by a single spontaneous source event, explicit formulas for $D_{G,\ell}, (\ell = 1, 2, 3)$ in the prompt, point kinetics approximation are given by:

$$
\begin{aligned}
D_{G,1}(U, M_L) &= (U(D_{sf,1} - 1) + 1)M_L \\
D_{G,2}(U, M_L) &= M_L^2\left(UD_{sf,2} + \frac{M_L - 1}{D_{if,1} - 1}(U(D_{sf,1} - 1) + 1)D_{if,2}\right) \\
D_{G,3}(U, M_L) &= M_L^3(UD_{sf,3} + \frac{M_L - 1}{D_{if,1} - 1}(3UD_{sf,2}U_{if,2} \\
&\quad + D_{if,3}(U(D_{sf,1} - 1) + 1)) \\
&\quad + 3\left(\frac{M_L - 1}{D_{if,1} - 1}\right)^2 D_{if,2}^2(U(D_{sf,1} - 1) + 1)).
\end{aligned}
\tag{2.2}
$$

Eqs. (2.1) and (2.2) form a set of three (non linear) equations with three unknowns. Once the set of equations is solved, the mass is proportional to $S \times U$, and the proportion coefficient is the reciprocal of the spontaneous fission rate (per gram). When measuring Pu samples, the spontaneous fission rate is approximately 473.5 fissions per gram per second [2].

2.2. Aim and motivation

In recent years, the use of NMC methods has seen constant growth, becoming a standard tool in safety, safeguards and facility operations. Thus, the need for a full uncertainty quantification is becoming more important. In response, we see growing interest, both academic and practical, in uncertainty quantification in NMC [9].

As stated, in the present study, we will restrict our discussions to the third factor only: statistical uncertainty. Quantification of the statistical uncertainty of the measurement variance in NMC, naturally, has been studied before, and there are several publications regarding both the

[1] The term "mathematically equivalent" refers to the fact that all methods share the same physical interpretation and model assumptions (and the same statistical convergence rate). But how the information is obtained may differ: different hardware, overlapping vs. non overlapping gates, different accidental estimations, different dead time formulation etc. The expectation for all methods will be the same even though the uncertainty might not.

[2] We use the notations $\widehat{E}(X), \widehat{Var}(X)'\widehat{Sk}(X)$ rather that $E(X), Var(X), Sk(X)$ to distinct between the sampled moments, and the theoretical moments, as would be sampled in a infinite measurement.

[3] If we denote by S_f the spontaneous fission rate, and by S_α the (α, n) rate, then $S = S_f + S_\alpha$ and $U = S_f/(S_f + S_\alpha)$.